

Zero Knowledge from MPC

CS 598 DH

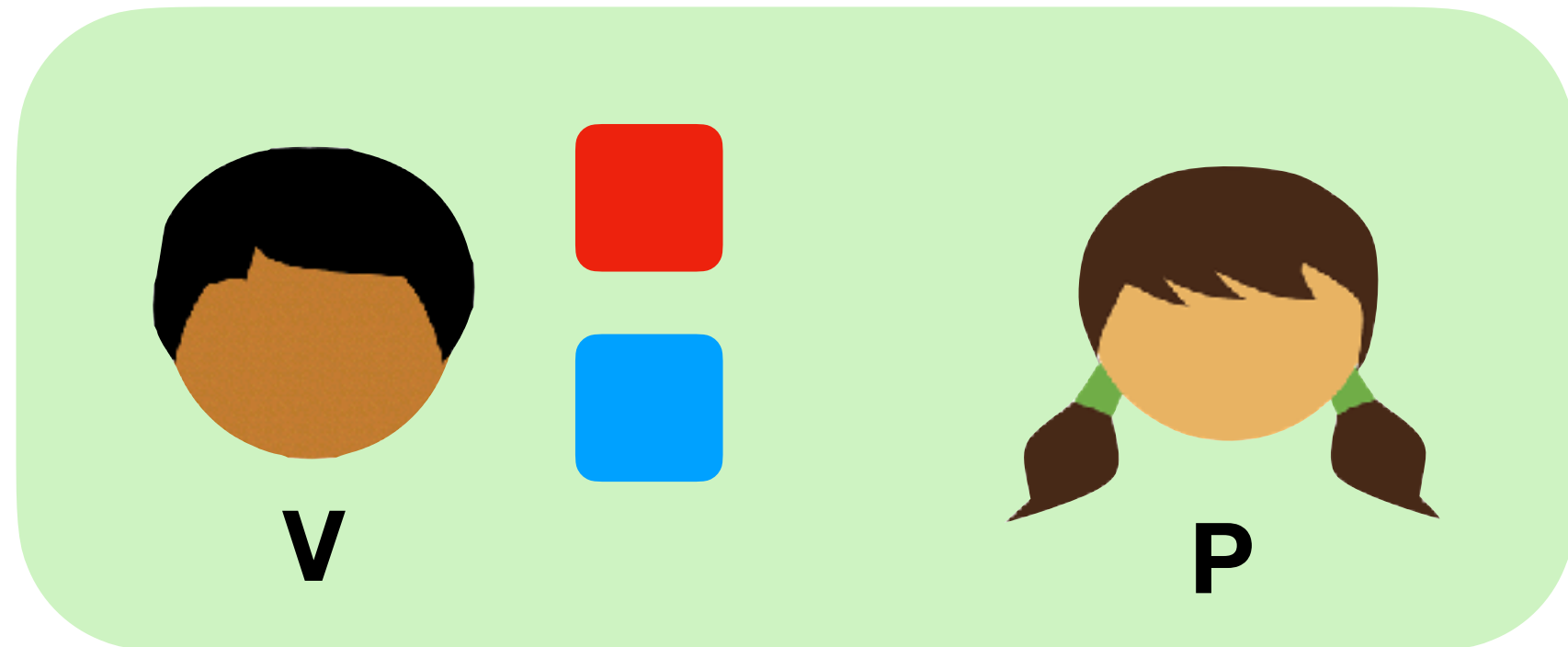
Today's objectives

Review Zero Knowledge Proofs

Construct ZK for circuit satisfiability problem

- From Garbled Circuits
- Via “MPC in the Head”

What is a *zero-knowledge* proof?



Completeness: If $x \in \mathcal{L}$ and if P and V are honest, then V accepts the proof (except with negligible probability)

“P can prove true things”

Soundness: If $x \notin \mathcal{L}$, even malicious P cannot cause honest V to accept the proof

“P cannot prove false things”

Zero Knowledge: “V learns nothing except that the thing is true”

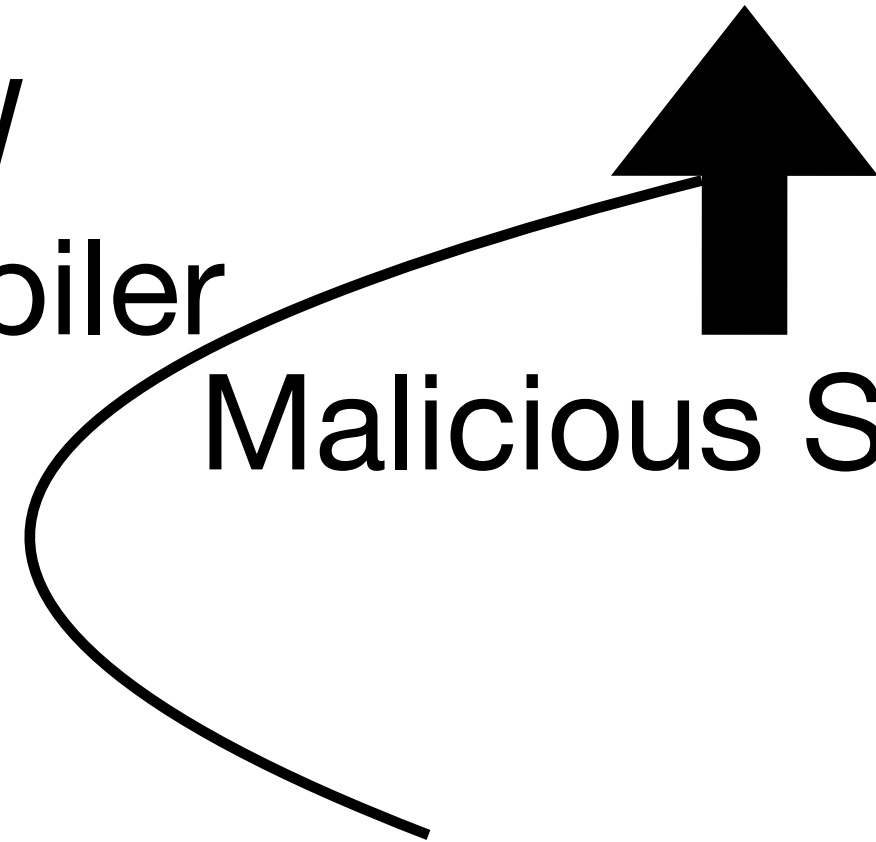
Setting

Semi-honest Security

GMW
Compiler

Malicious Security

Zero Knowledge



General-Purpose Tools

GMW Protocol

Multi-party

Multi-round

Garbled Circuit

Constant Round

Two Party

Primitives

Oblivious Transfer

Pseudorandom functions/encryption

Commitments

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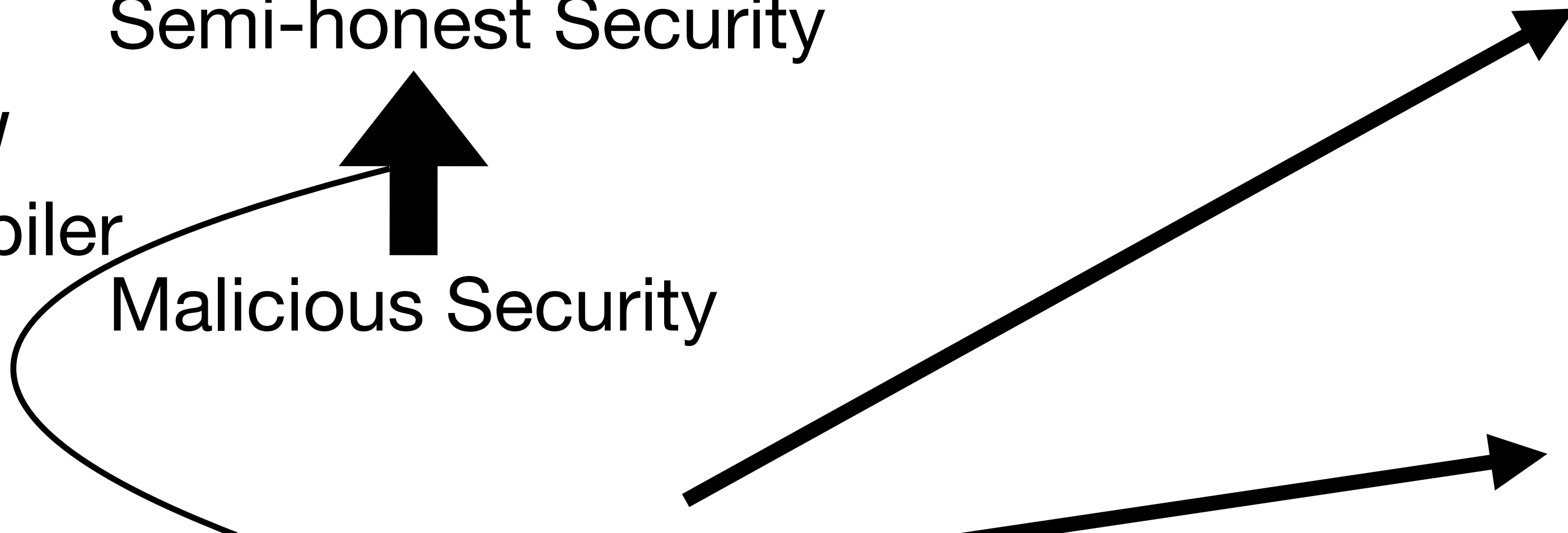
Two Party

Primitives

Oblivious Transfer

Pseudorandom functions/encryption

Commitments



Zero Knowledge Proofs of Circuit Satisfiability

Zero-Knowledge Using Garbled Circuits

or How To Prove Non-Algebraic Statements Efficiently*

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ABSTRACT

Zero-knowledge protocols are one of the fundamental concepts in modern cryptography and have countless applications. However, after more than 30 years from their introduction, there are only very few languages (essentially those with a group structure) for which we can construct zero-knowledge protocols that are efficient enough to be used in practice.

In this paper we address the problem of how to construct efficient zero-knowledge protocols for generic languages and we propose a protocol based on Yao's garbled circuit technique.

The motivation for our work is that in many cryptographic applications it is useful to be able to prove efficiently statements of the form e.g., "I know x s.t. $y = \text{SHA-256}(x)$ " for a common input y (or other "unstructured" languages), but no efficient protocols for this task are currently known.

It is clear that zero-knowledge is a subset of secure two-party computation (i.e., any protocol for generic secure computation can be used to do zero-knowledge). The main contribution of this paper is to construct an efficient protocol for the special case of secure two-party computation where only one party has input (like in the zero-knowledge case). The protocol achieves active security and is essentially only twice as slow as the passive secure version of Yao's garbled circuit protocol. This is a great improvement with respect to the *cut-n-choose* technique to make Yao's protocol actively secure, where the complexity grows linearly with the security parameter.

Categories and Subject Descriptors

D.4.6 [Operating Systems]: Security and Protection—*Cryptographic controls*; C.2.4 [Computer-Communication Networks]: Distributed Systems—*distributed applications*

*A full version of this paper is available at [JKO13].

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<http://dx.doi.org/10.1145/2508859.2516662>.

Keywords

Zero-Knowledge Proof; Garbled Circuits; Efficiency

1. INTRODUCTION

Zero-knowledge (ZK) protocols have countless applications in cryptography and therefore efficiency is of paramount importance. Consequently, a huge effort has been put into designing efficient ZK protocols for *specific tasks*. In particular there are very efficient protocols for languages with some algebraic structure. There are, for instance, efficient protocols for proving knowledge and relations of discrete logarithms [Sch89, CDS94], for proving that RSA public keys are well formed [CM99], for statements in post-quantum cryptography [BD10, JKPT12, MV03], for bilinear equations [GS12, GSW10], for shuffles [BG12, KMW12] and frameworks for modular design of zero-knowledge protocols [CKS11].

However, *generic constructions* for ZK protocols use Karp reductions to NP-complete languages and are therefore too impractical to be used in practice. In particular, so far there has been no practical solution to problems that do not exhibit an algebraic structure. Examples for protocols that could be used in many cryptographic applications are e.g., the problem of efficiently proving statements of the form "I know x s.t. $y = \text{SHA-256}(x)$ " or "I know k s.t. $y_1 = \text{AES}_k(y_2)$ " (the common input is y in the first example and (y_1, y_2) in the second)¹.

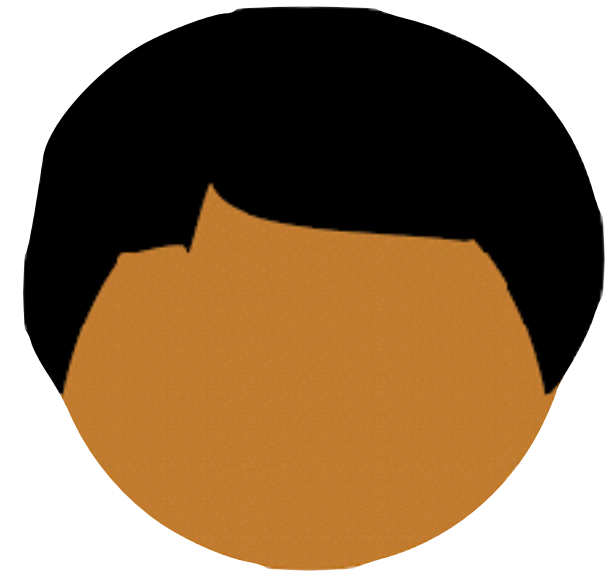
In this work we provide a *generic and efficient* solution for proving any such statements in zero-knowledge, by constructing a protocol based on Yao's garbled circuits technique. The complexity of our protocol is proportional to the size of the circuit of the NP verification function. To support the validity of our efficiency claim, we present also a *proof-of-concept* implementation of our protocol. The performance measurements of our prototypical implementation show the viability of our protocol for realistic problems.

1.1 Zero-Knowledge and 2PC

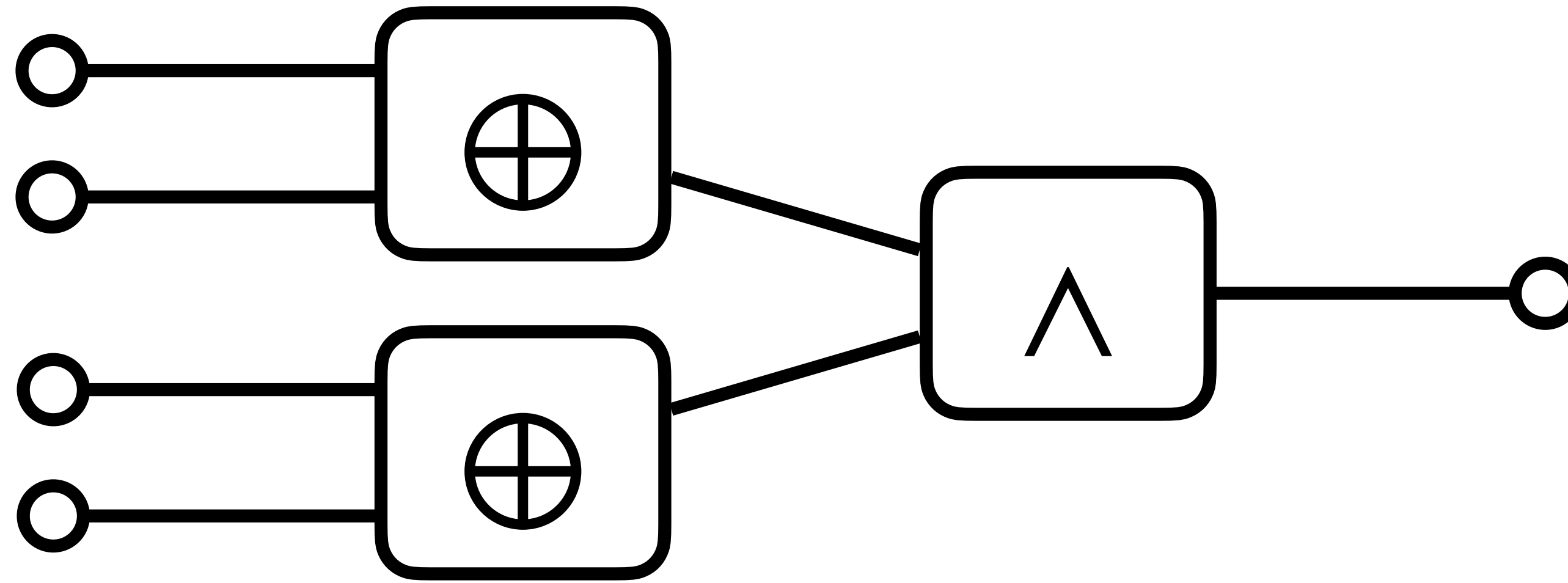
Zero-knowledge proofs were introduced more than 30 years ago by Goldwasser, Micali and Rackoff [GMR85]. A zero-knowledge argument (ZK) is an interactive protocol that allows a prover P to persuade a verifier V of the validity of some NP statement y by using the knowledge of a witness w . Informally, an honest prover should be able to

¹Note that in both cases the prover is not only showing that the instance belongs to the language (both languages are trivial), but moreover that the prover *knows* a valid witness for this. So these proofs are meaningful as we believe that it is hard to compute such a witness.

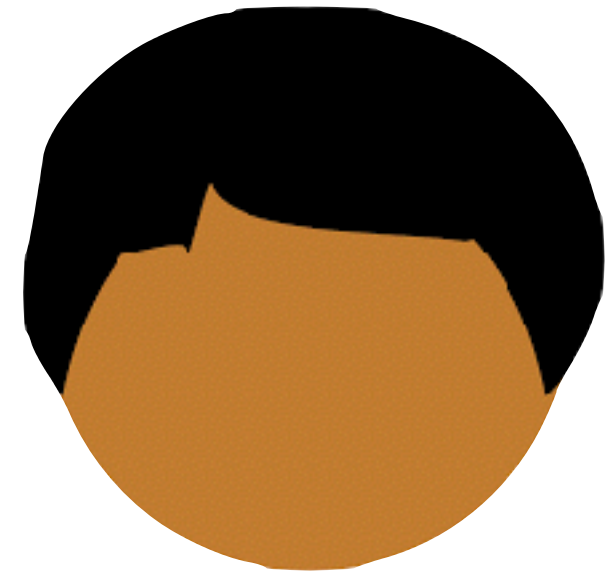
Refresher: Garbled Circuits



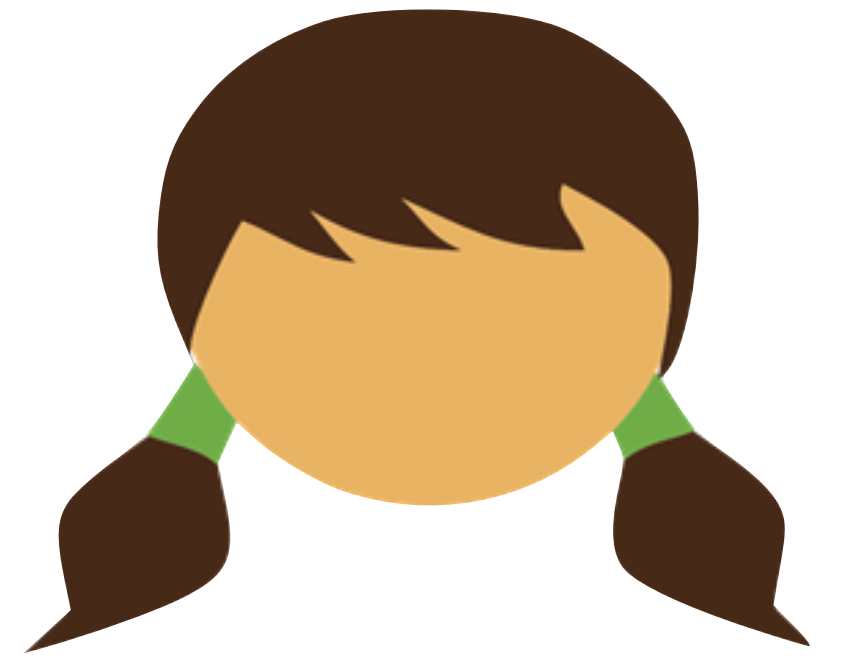
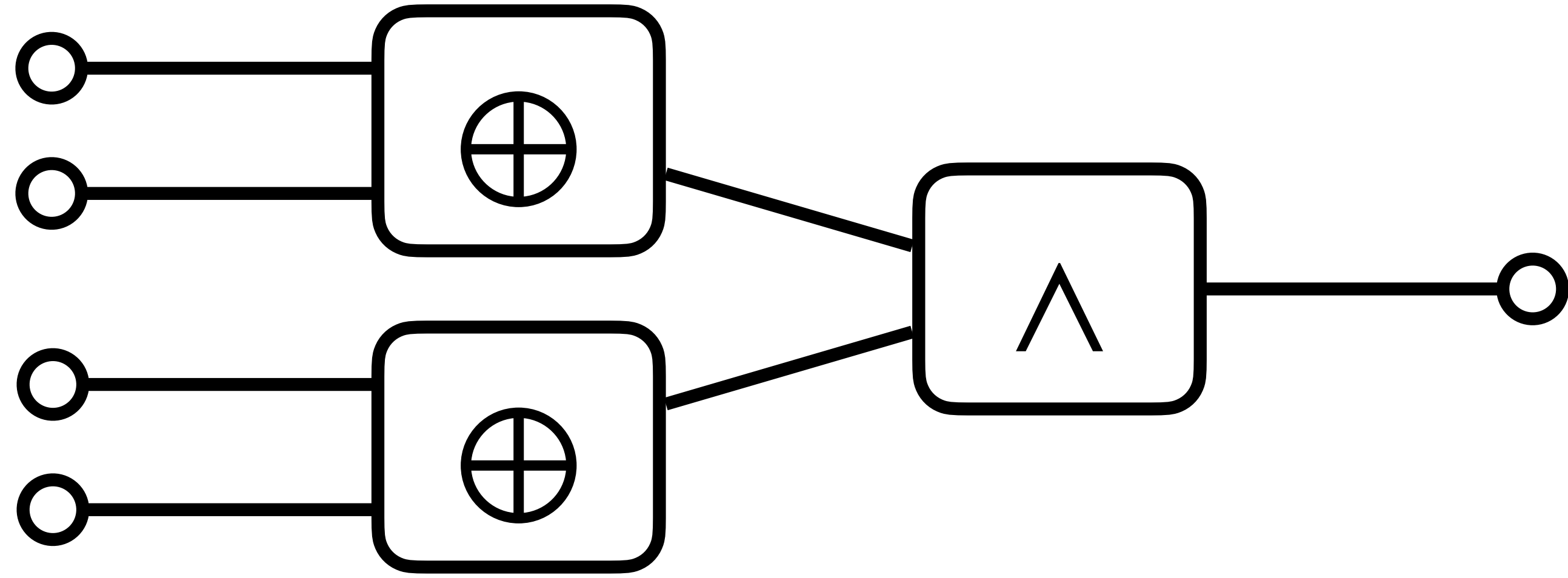
Garbler



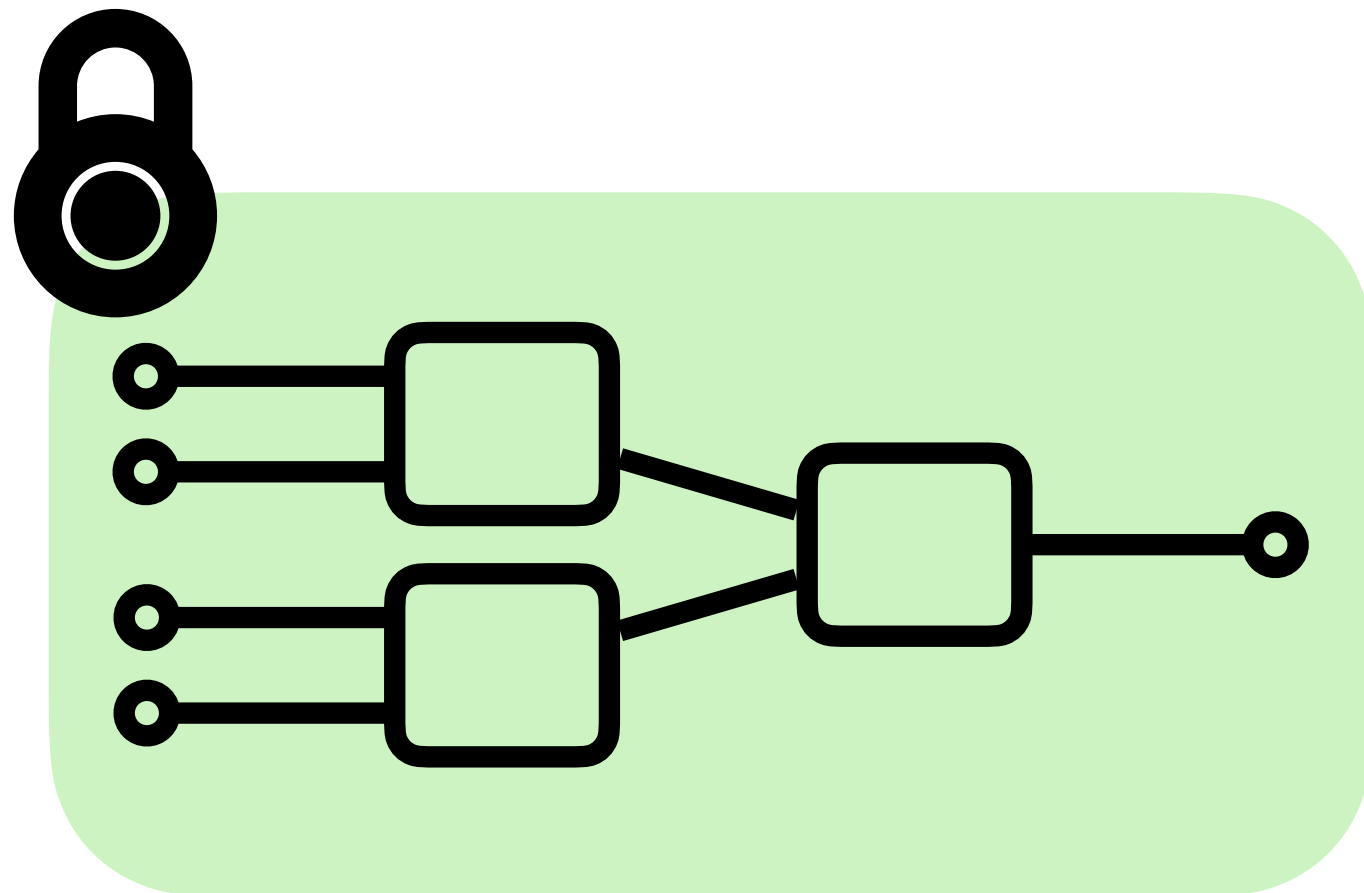
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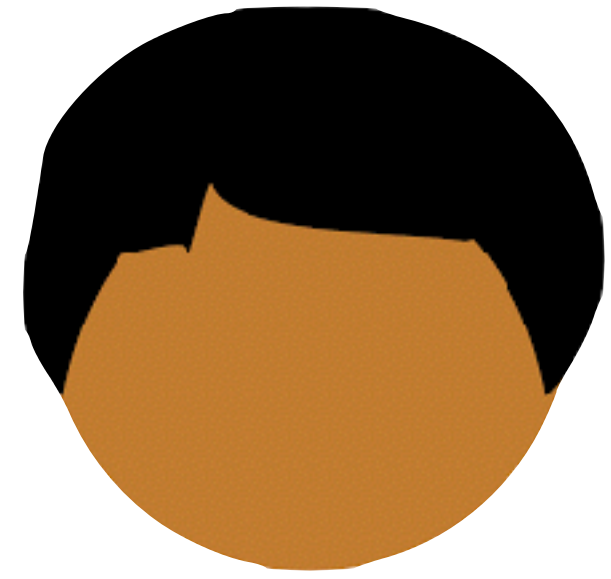


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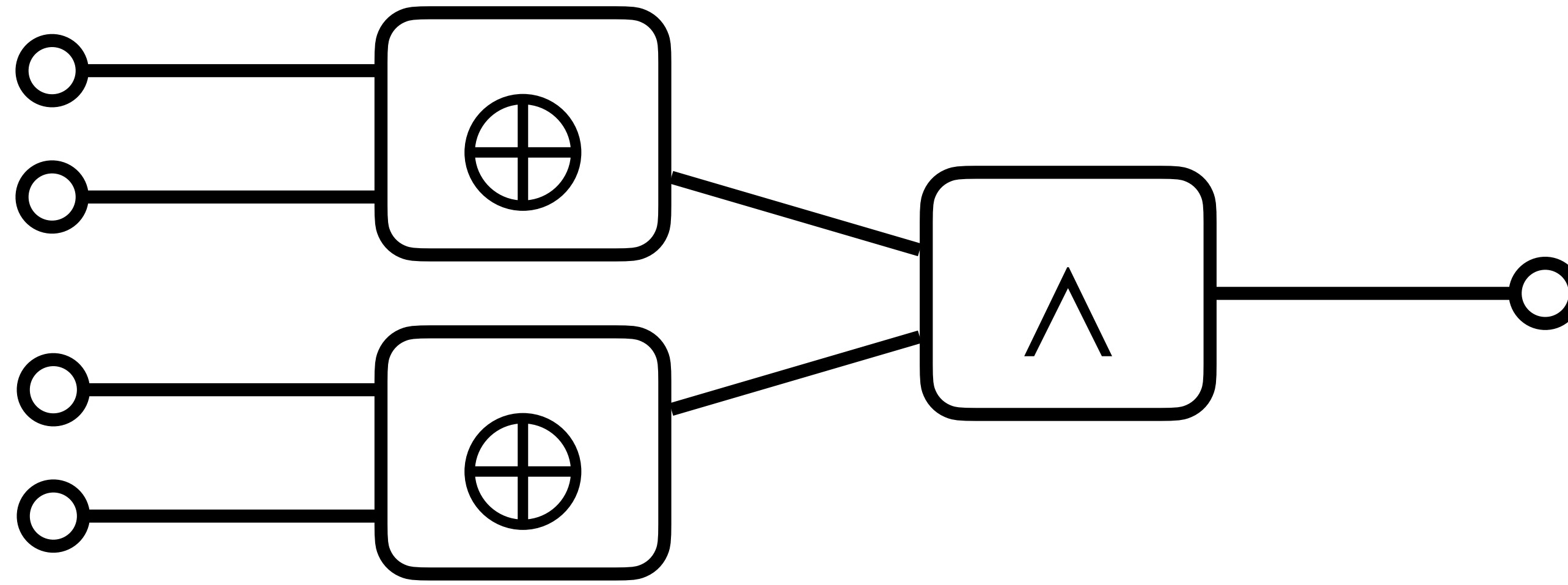


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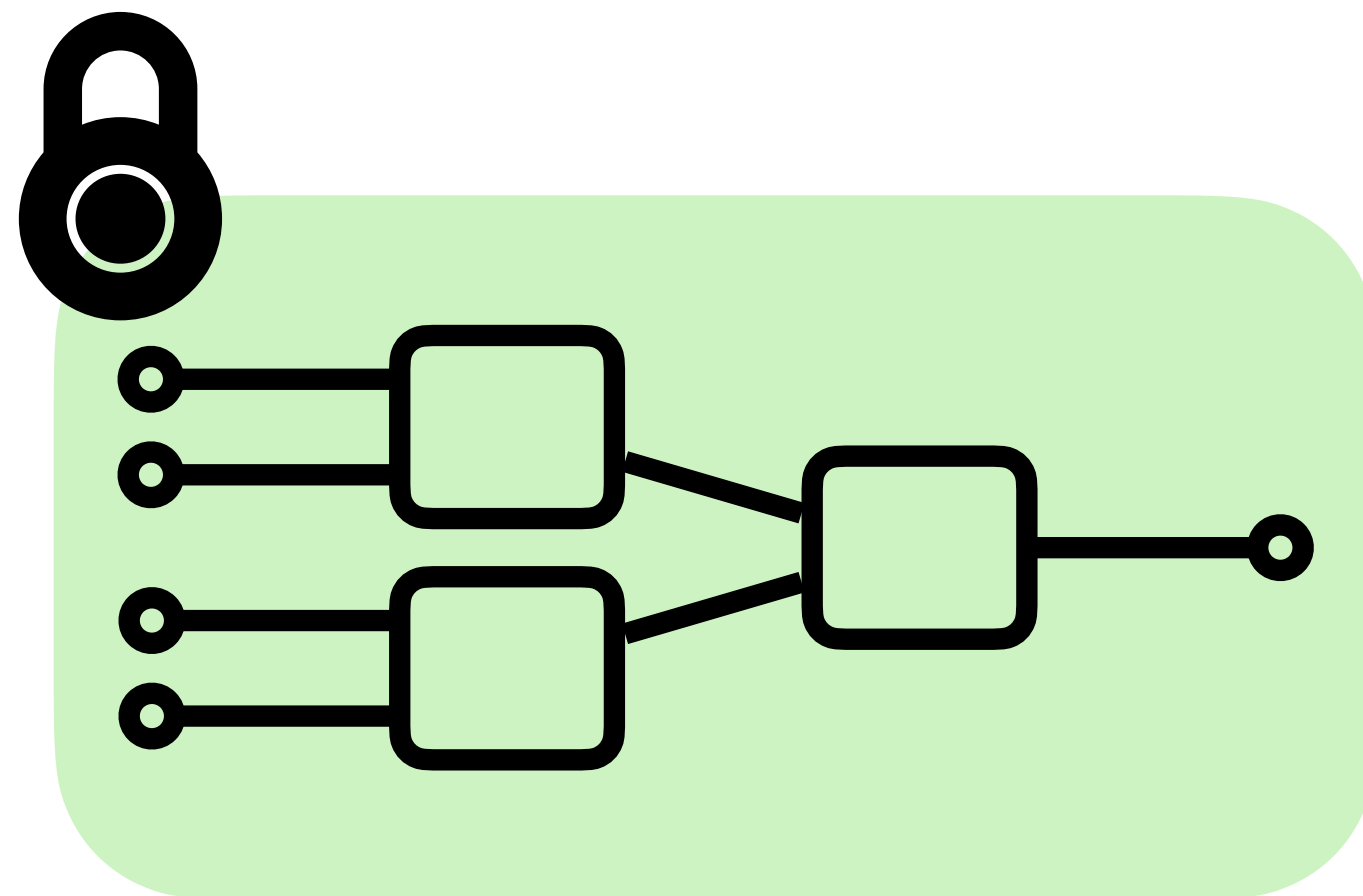


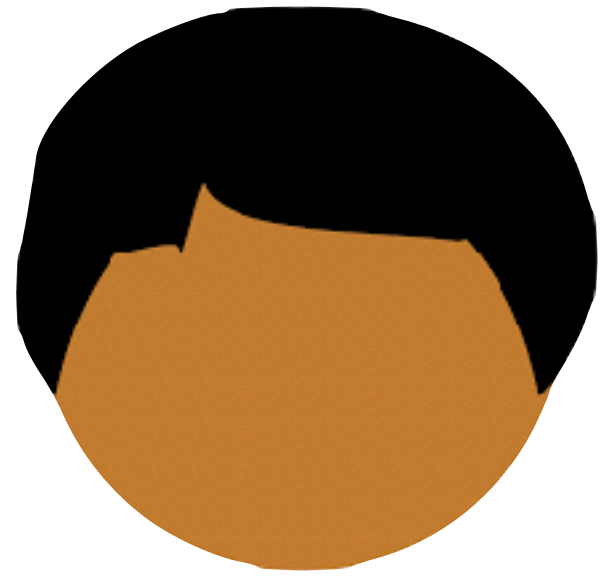


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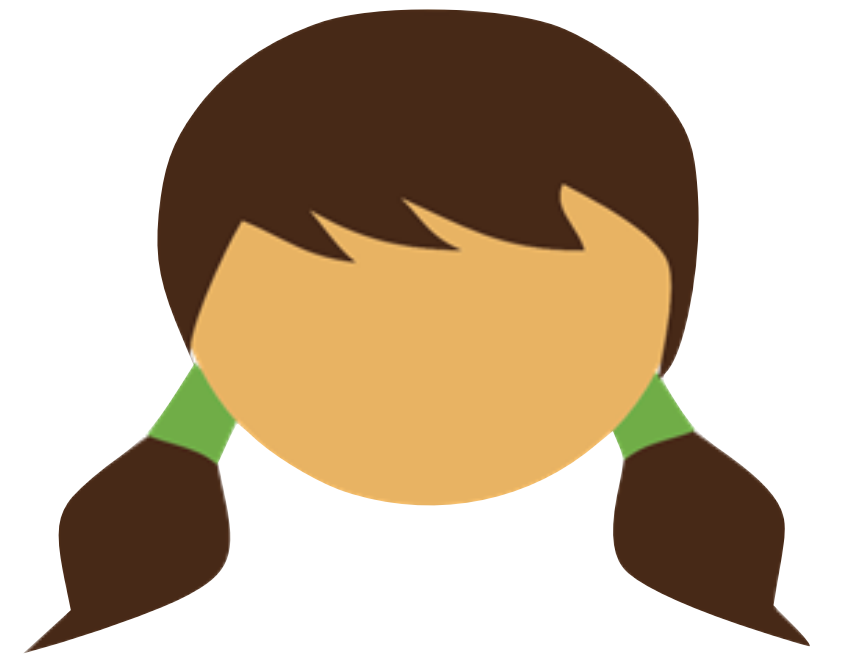
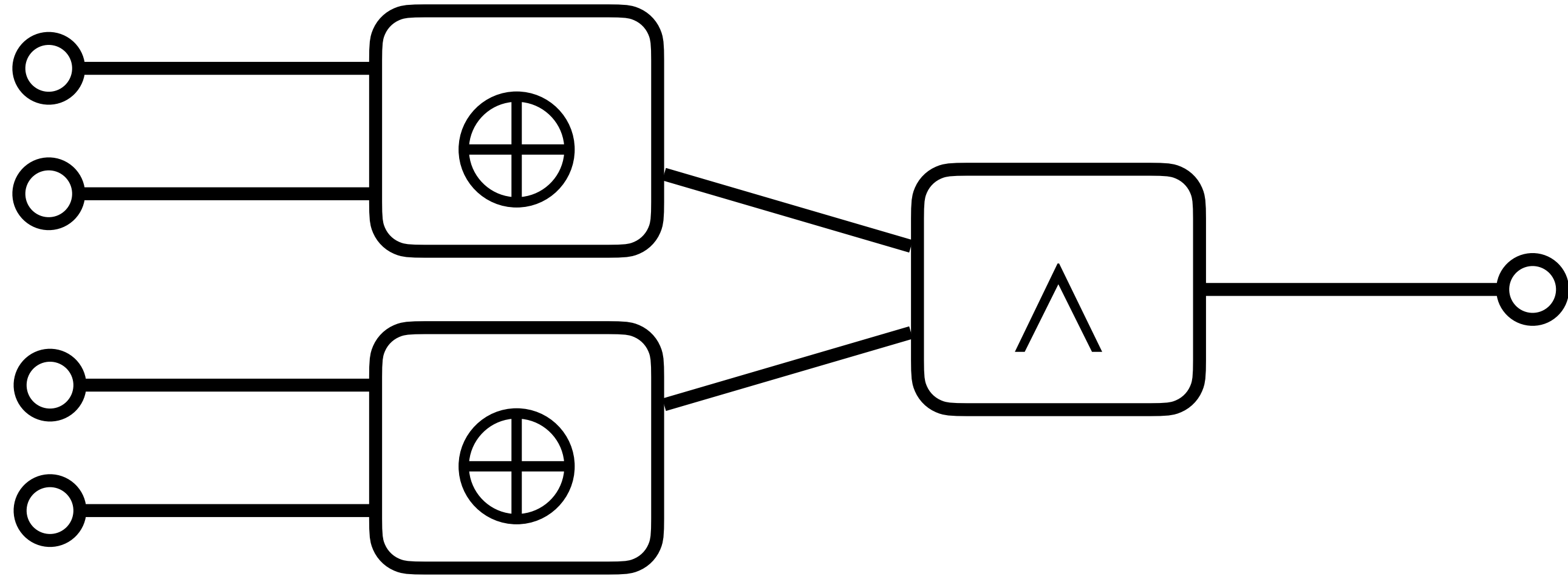


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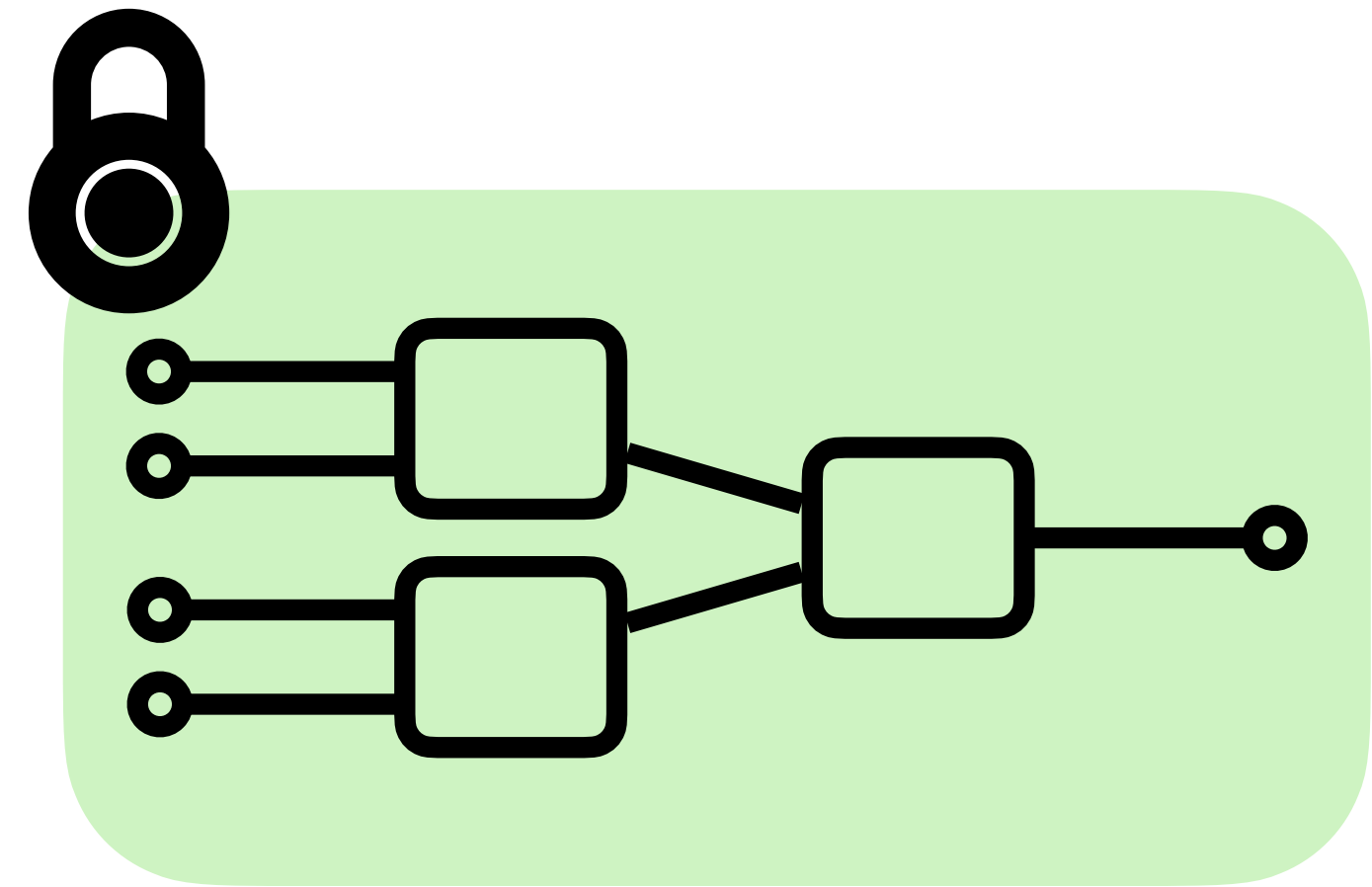
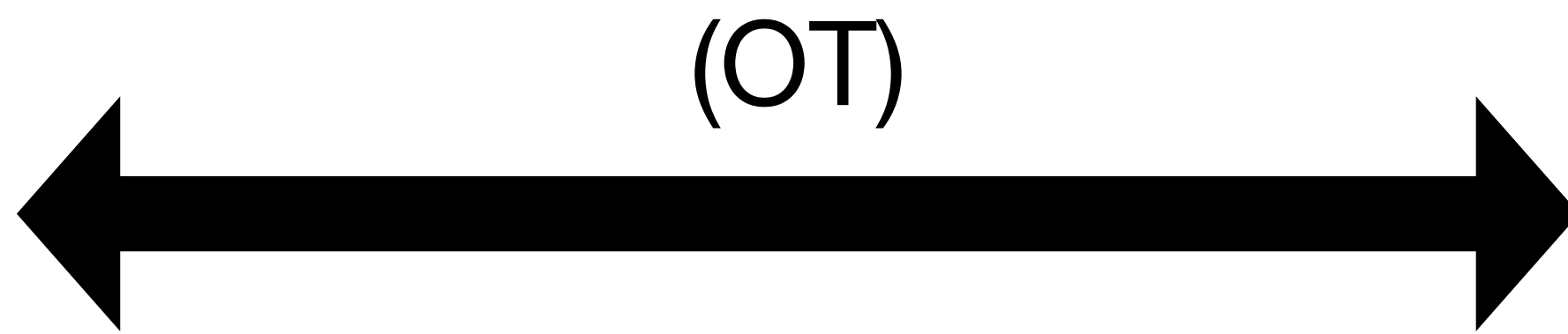


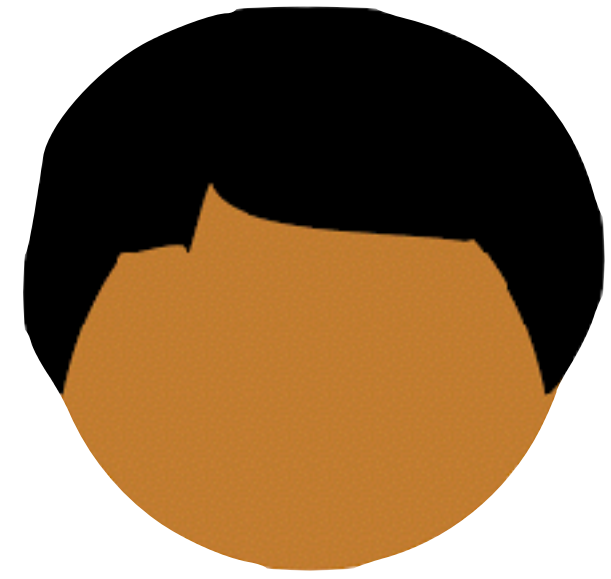


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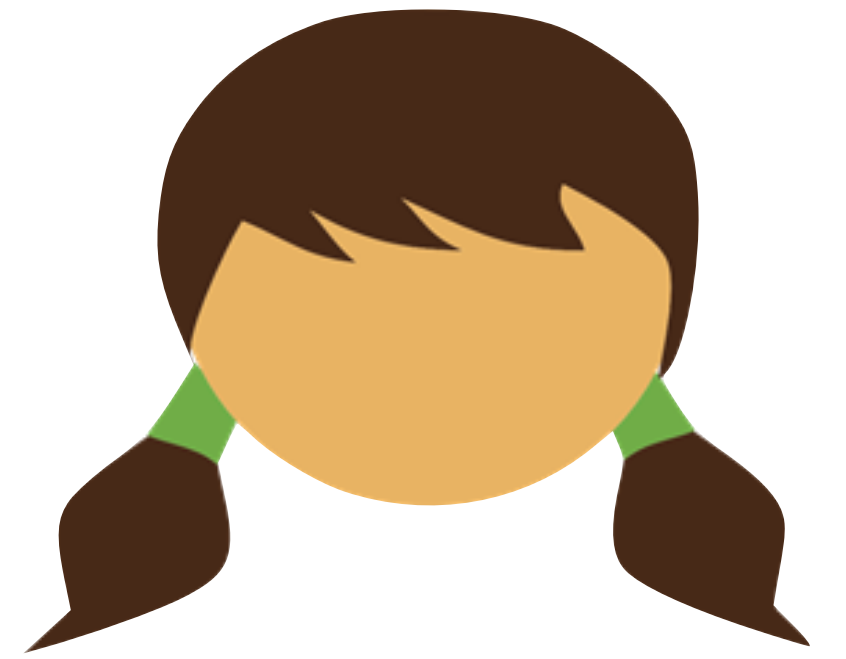
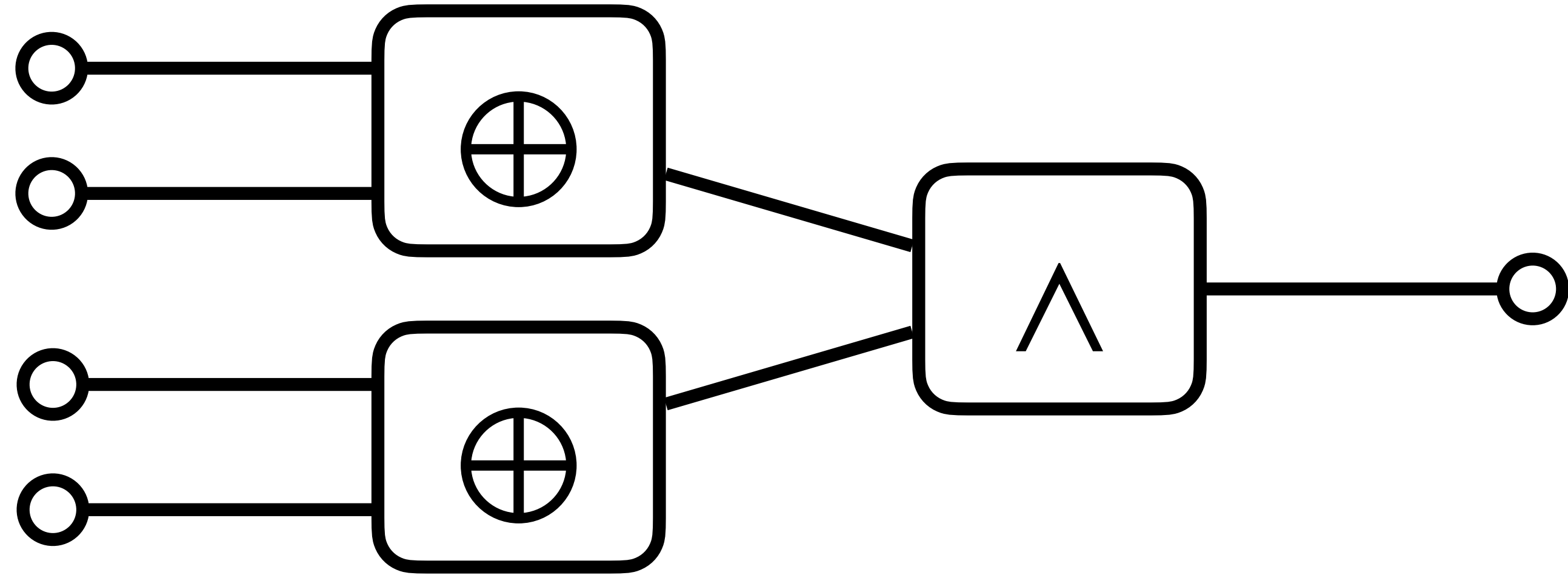


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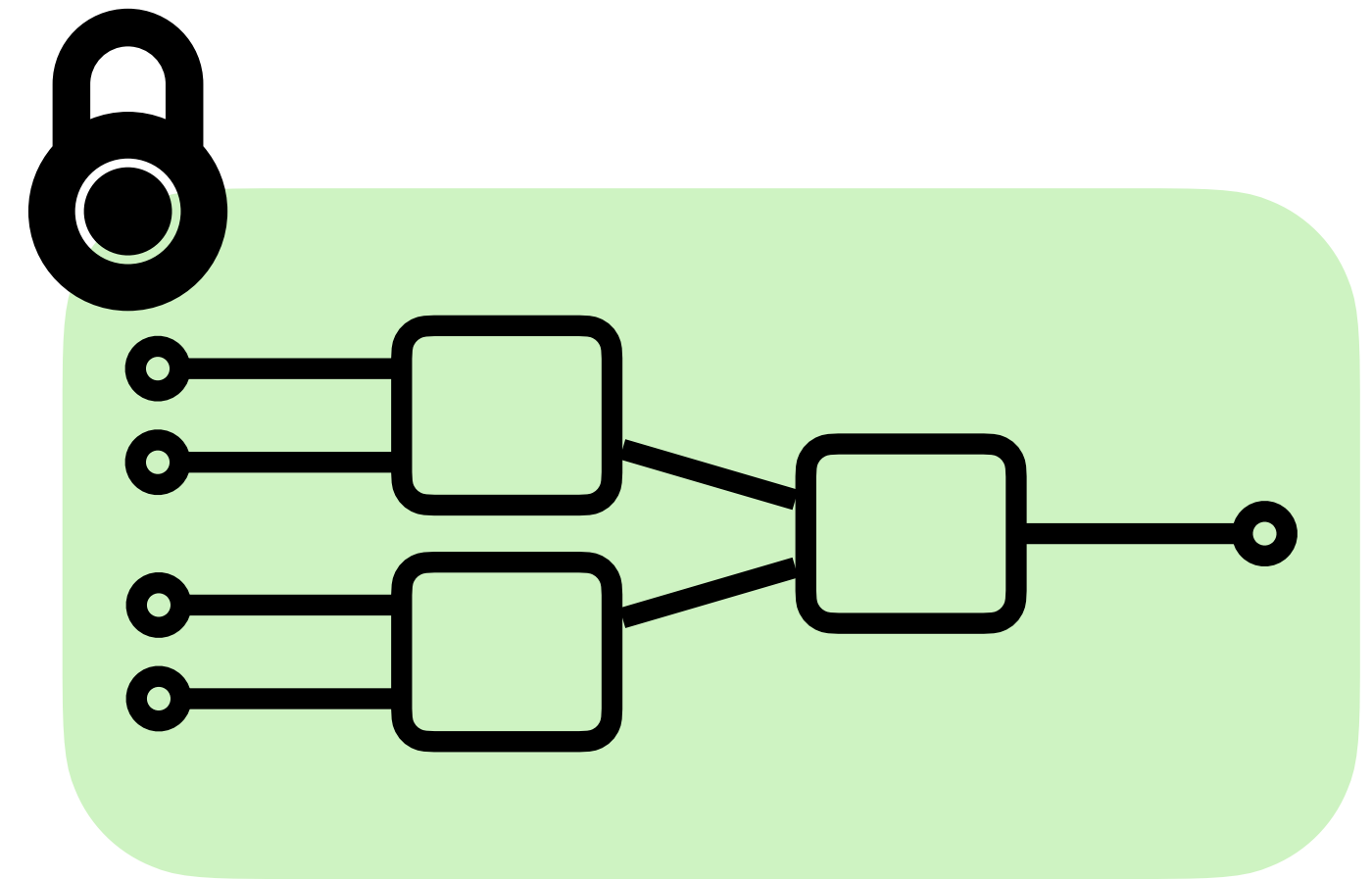


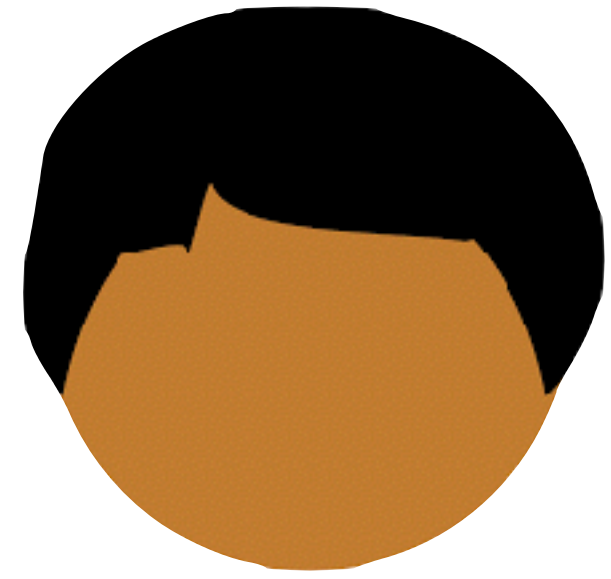


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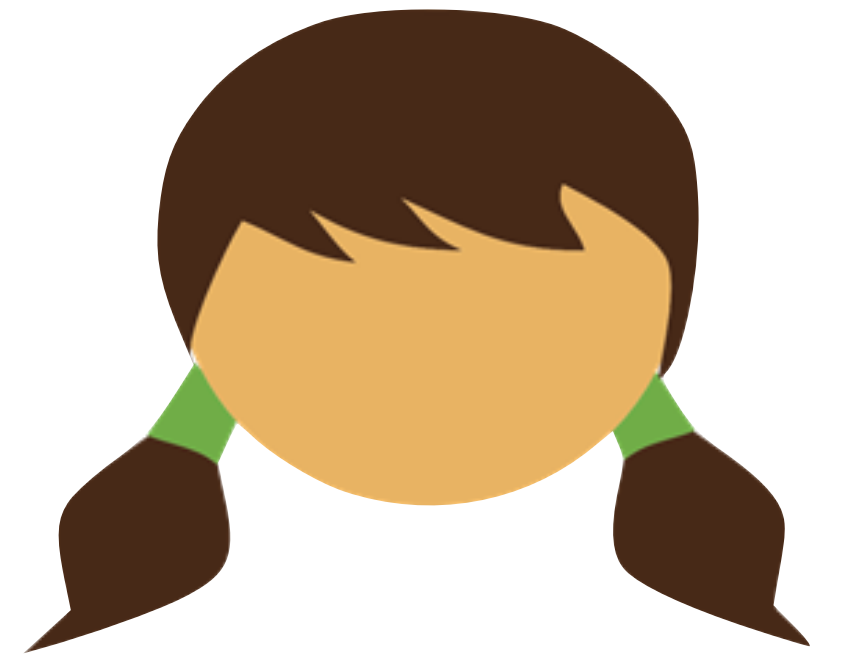
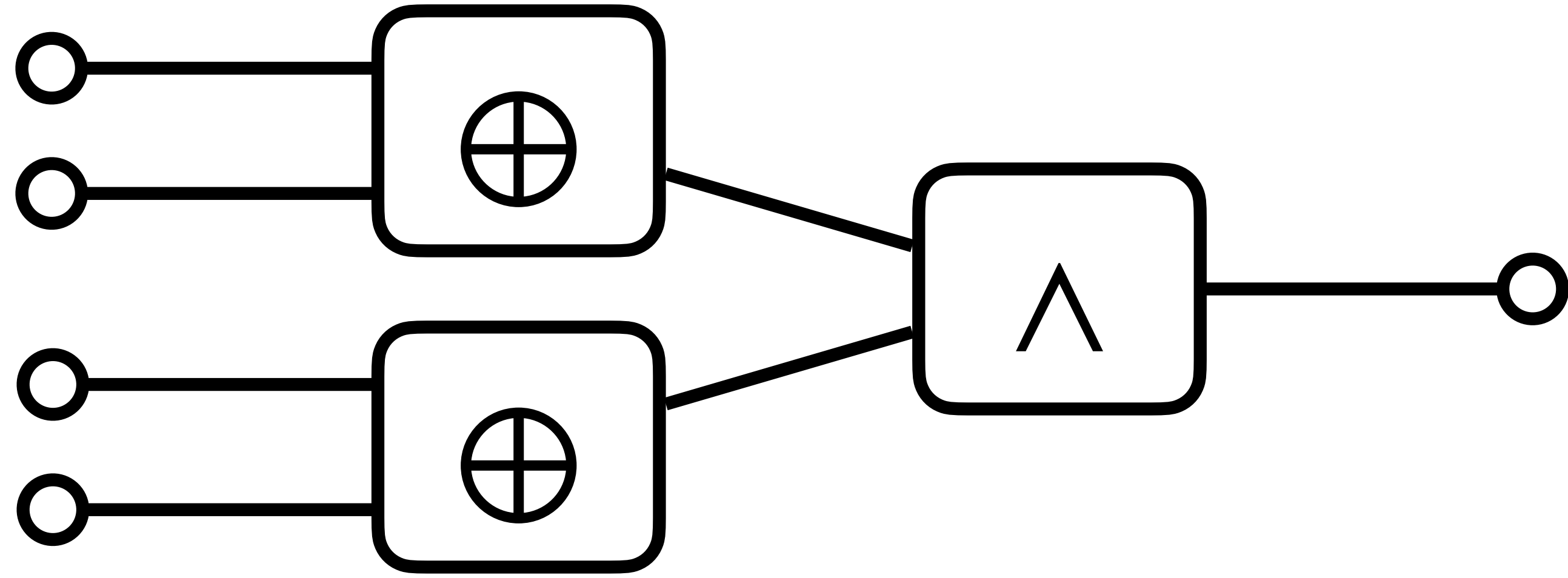


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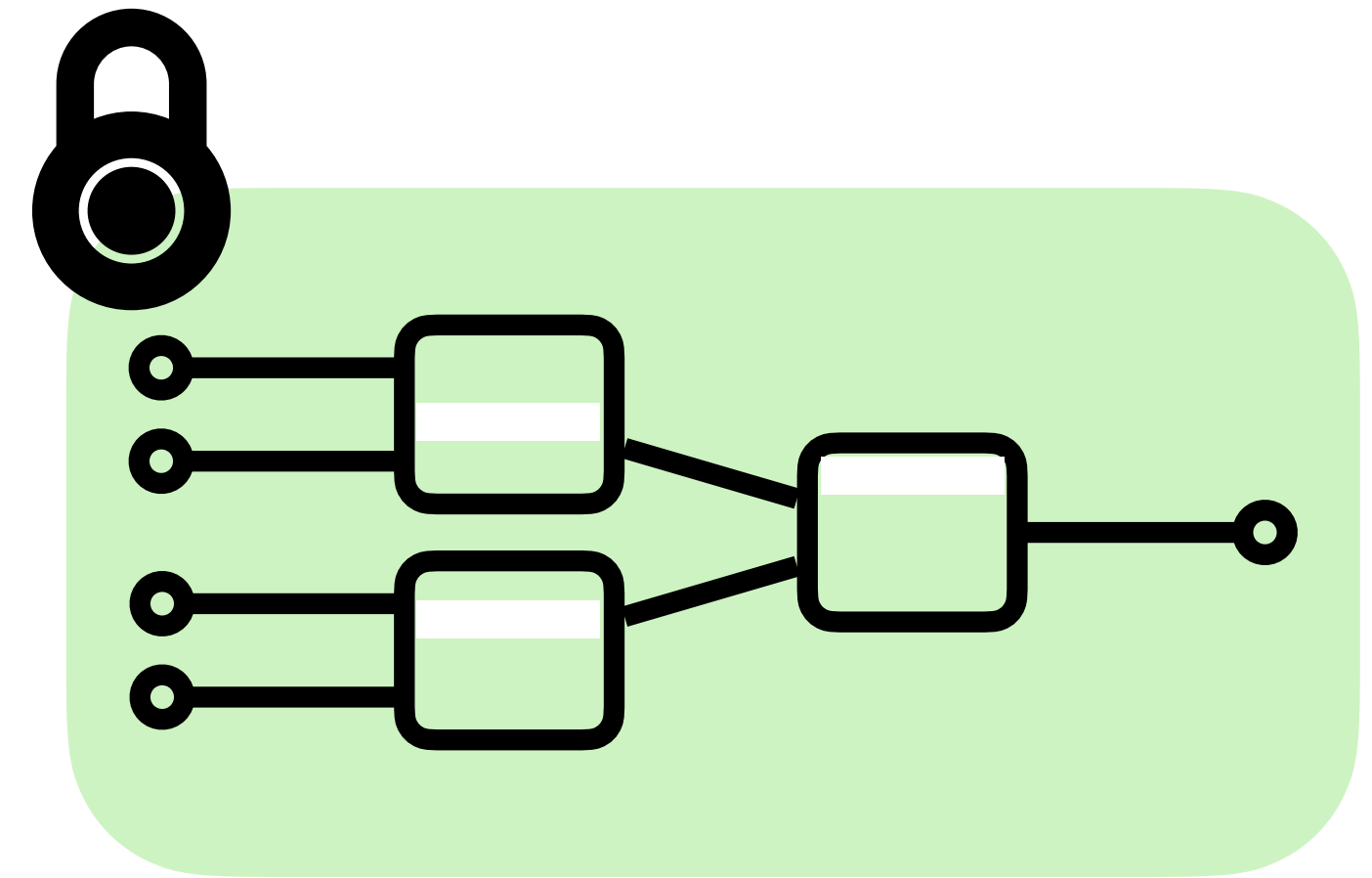


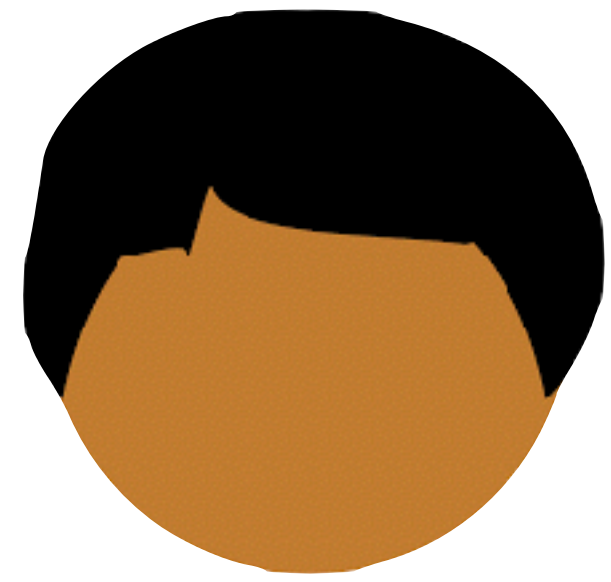


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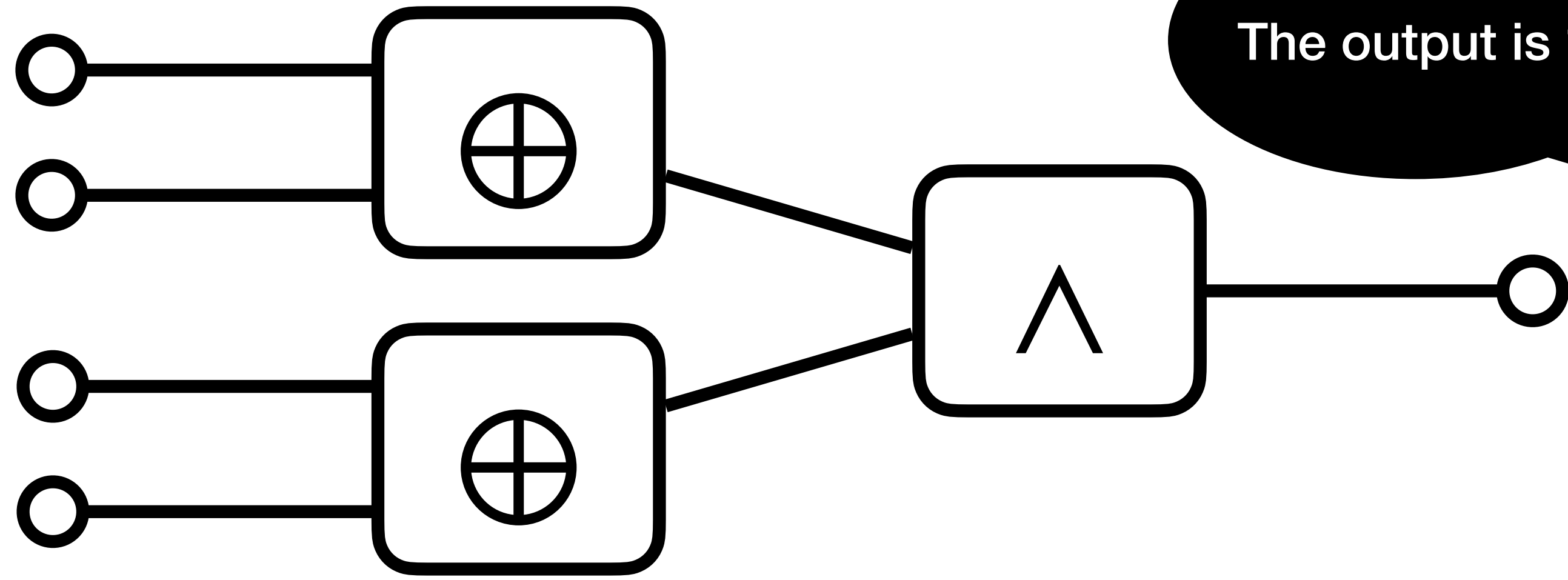


Evaluator





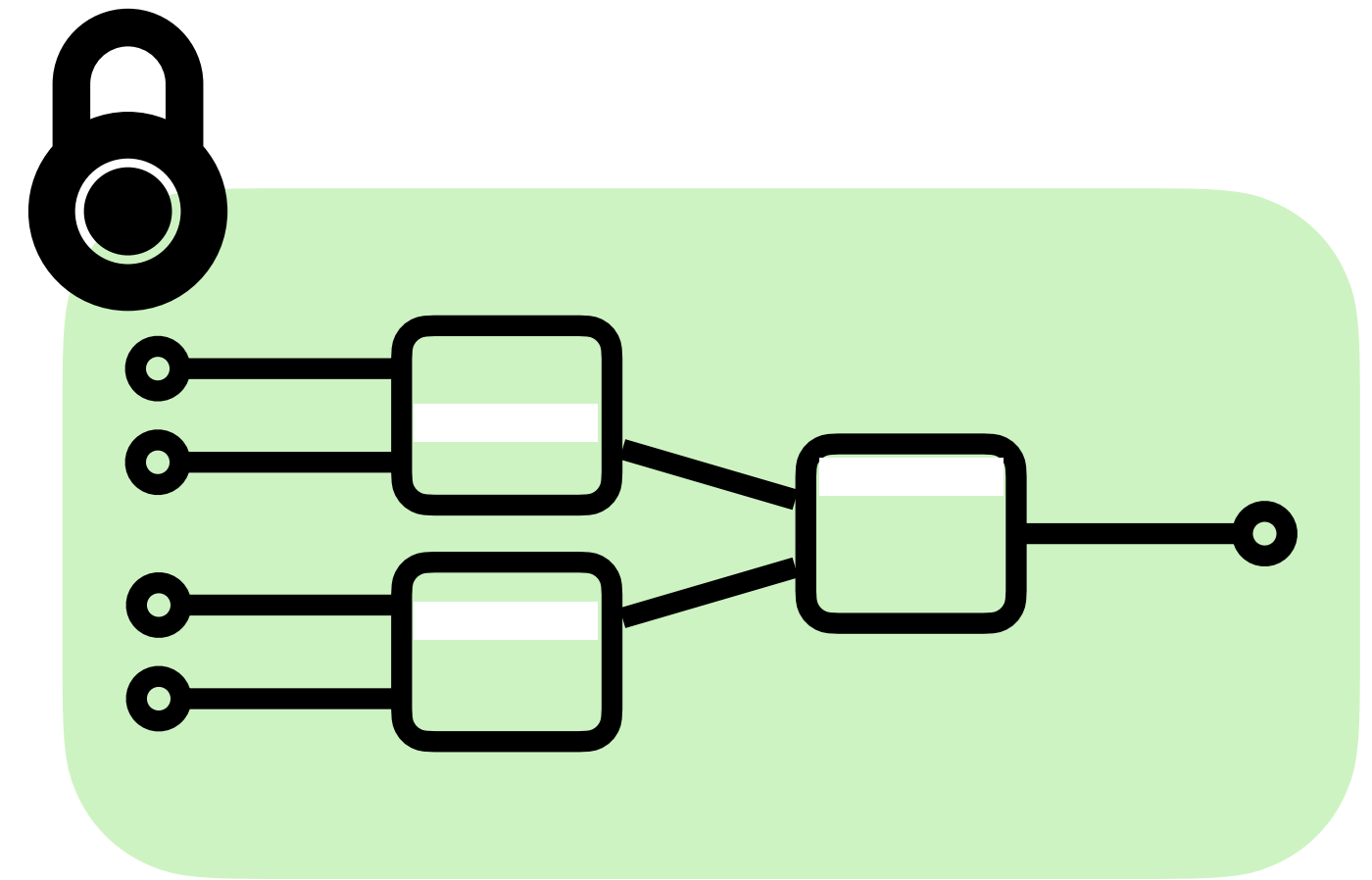
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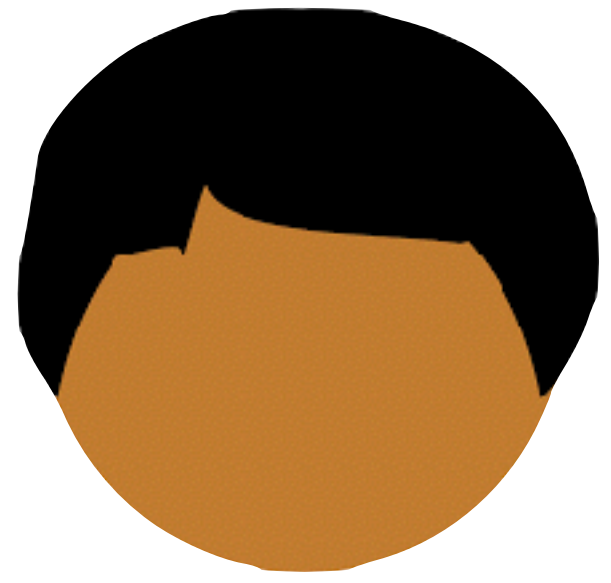


The output is 1

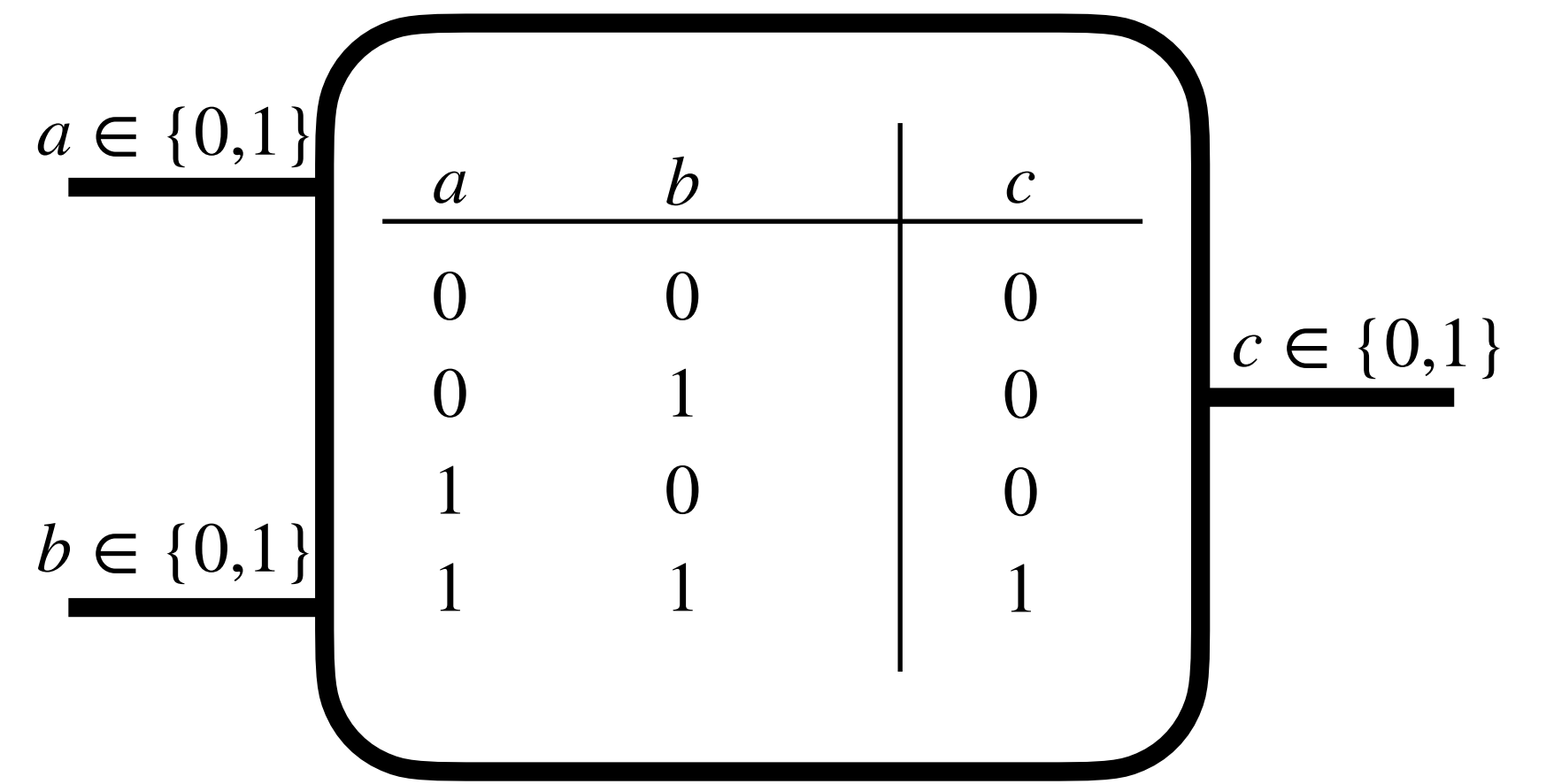


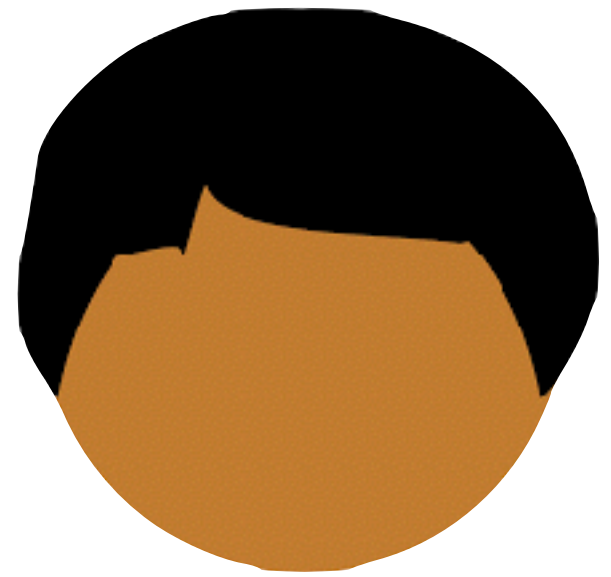
Evaluator





G





G

$a \in \{K_a^0, K_a^1\}$

$b \in \{K_b^0, K_b^1\}$

a	b	c
K_a^0	K_b^0	K_c^0
K_a^0	K_b^1	K_c^0
K_a^1	K_b^0	K_c^0
K_a^1	K_b^1	K_c^1

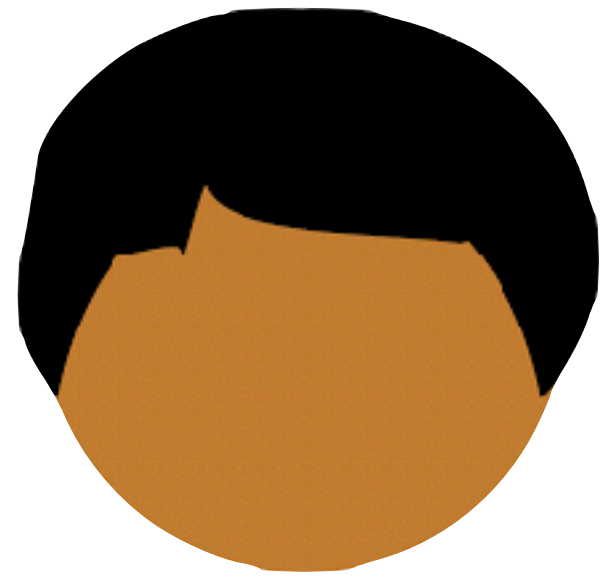
$c \in \{K_c^0, K_c^1\}$

$a \in \{0,1\}$

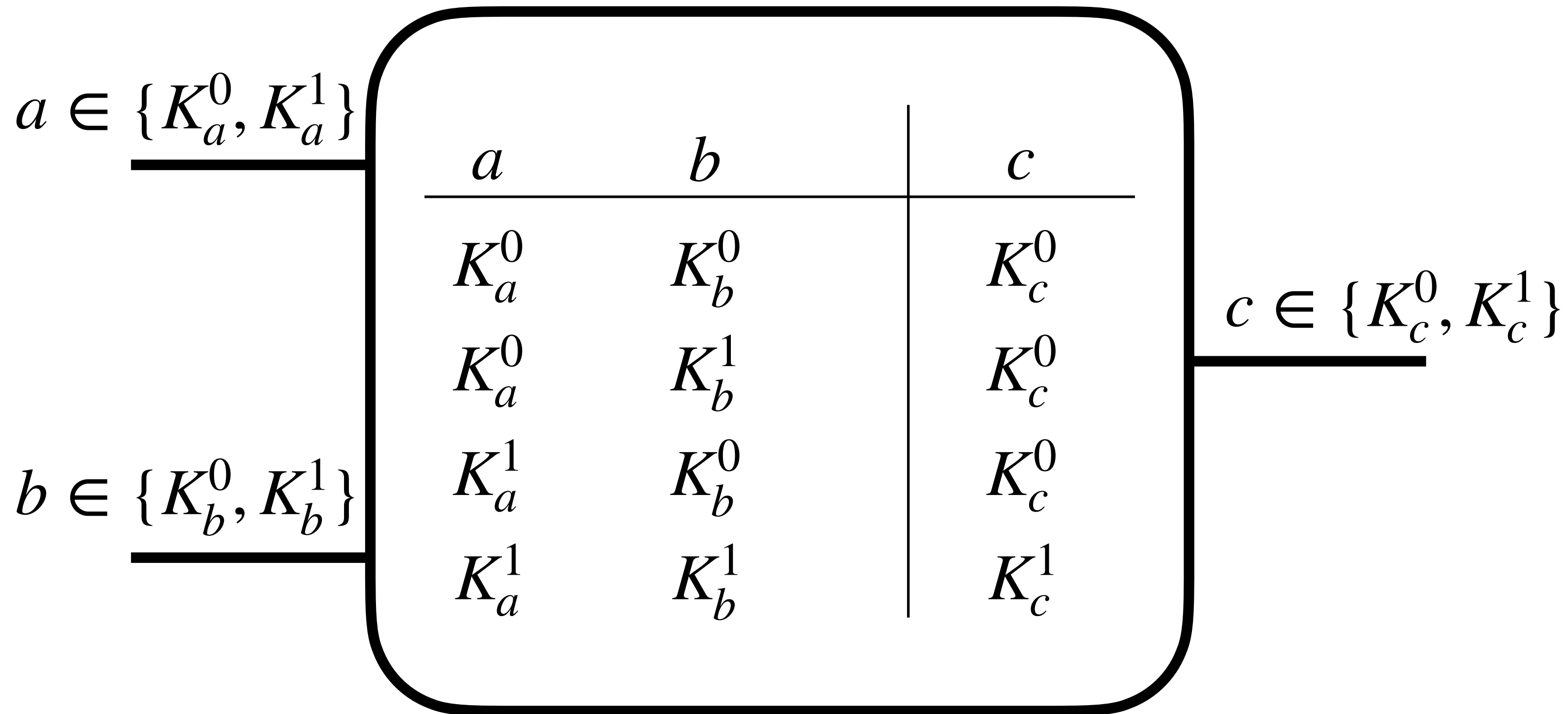
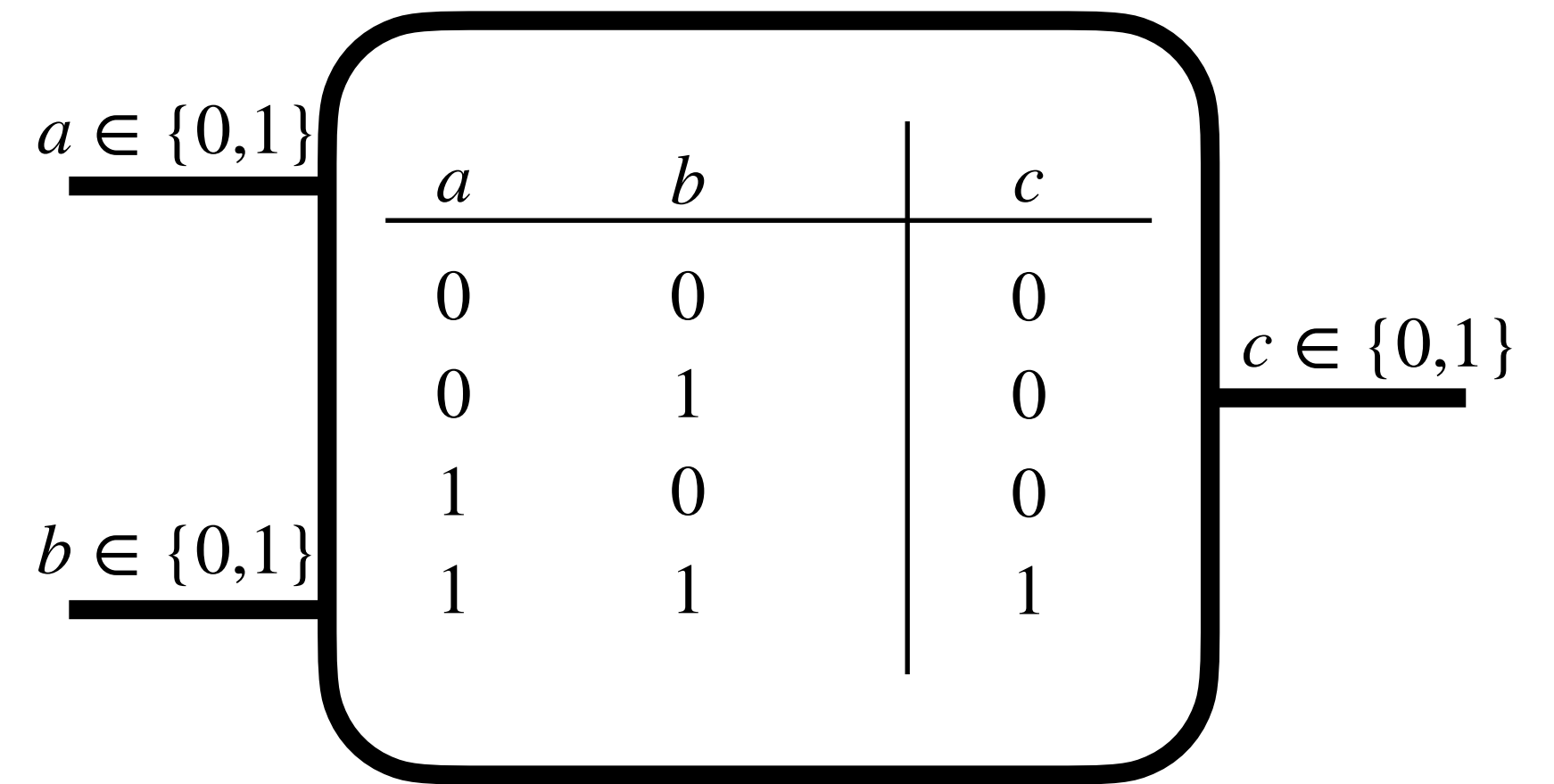
$b \in \{0,1\}$

a	b	c
0	0	0
0	1	0
1	0	0
1	1	1

$c \in \{0,1\}$



G

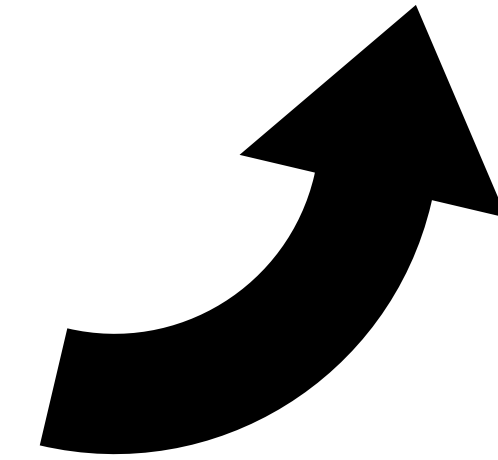


- $\text{Enc}(K_a^0, \text{Enc}(K_b^0, K_c^0))$
- $\text{Enc}(K_a^0, \text{Enc}(K_b^1, K_c^0))$
- $\text{Enc}(K_a^1, \text{Enc}(K_b^0, K_c^0))$
- $\text{Enc}(K_a^1, \text{Enc}(K_b^1, K_c^1))$

Zero Knowledge proof of circuit satisfiability

Proof system allows proofs of the form “*this circuit is satisfiable*”

There exists an input
s.t. the circuit outputs 1



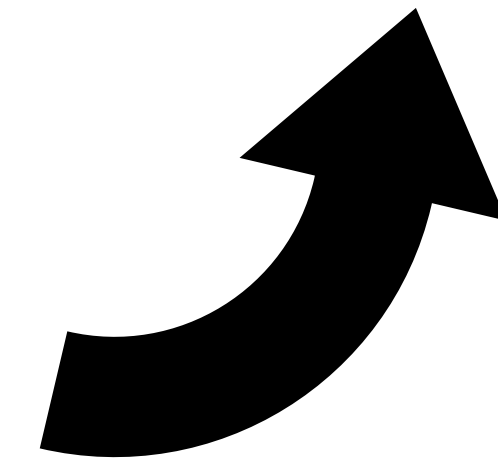
Statement:

Witness:

Zero Knowledge proof of circuit satisfiability

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There exists an input
s.t. the circuit outputs 1



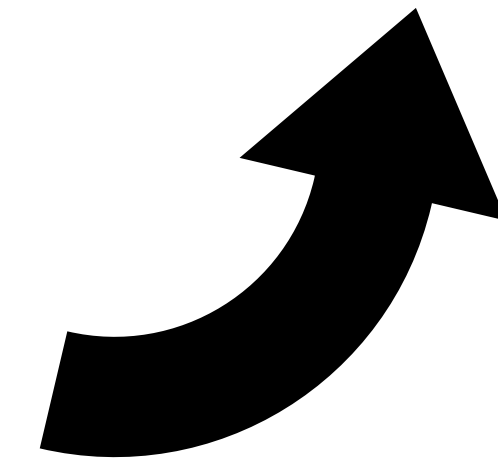
Statement: Boolean circuit C

Witness: A string x s.t. $C(x) = 1$

Zero Knowledge proof of circuit satisfiability

Proof system allows proofs of the form “*this circuit is satisfiable*”

There exists an input
s.t. the circuit outputs 1

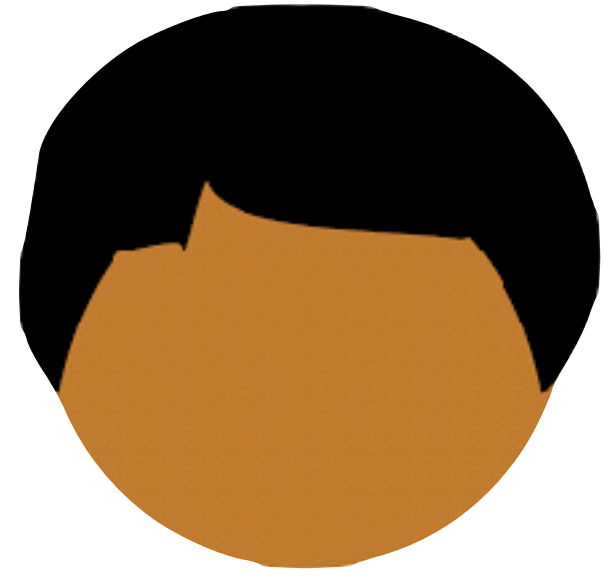


Statement: Boolean circuit C

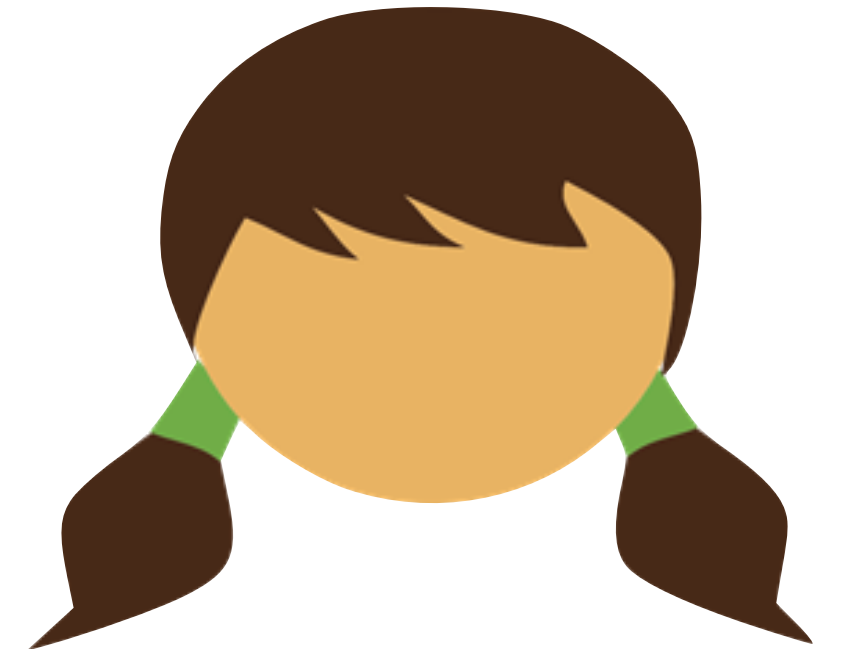
Witness: A string x s.t. $C(x) = 1$

It is relatively easy to compile arbitrary provable (NP) statements to circuits

Zero Knowledge from Garbled Circuits



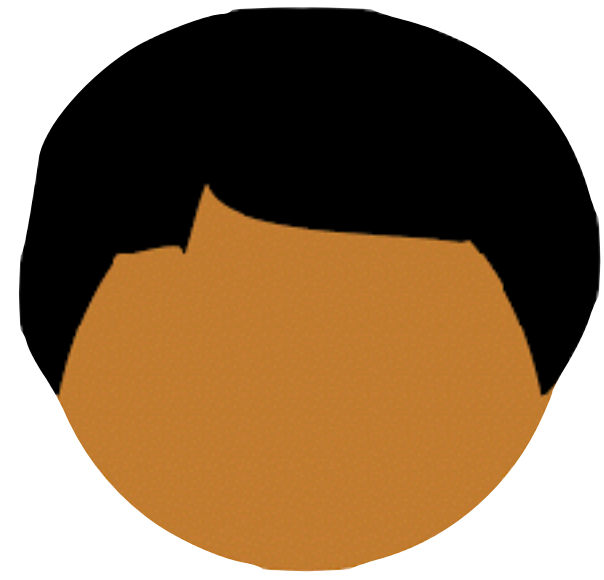
Verifier
Garbler



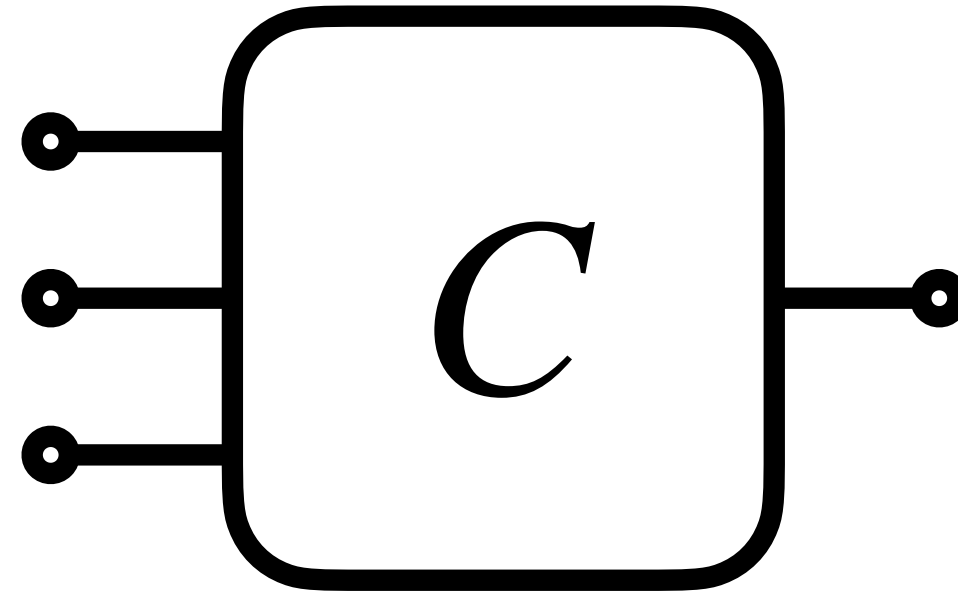
x

Prover
Evaluator

Zero Knowledge from Garbled Circuits



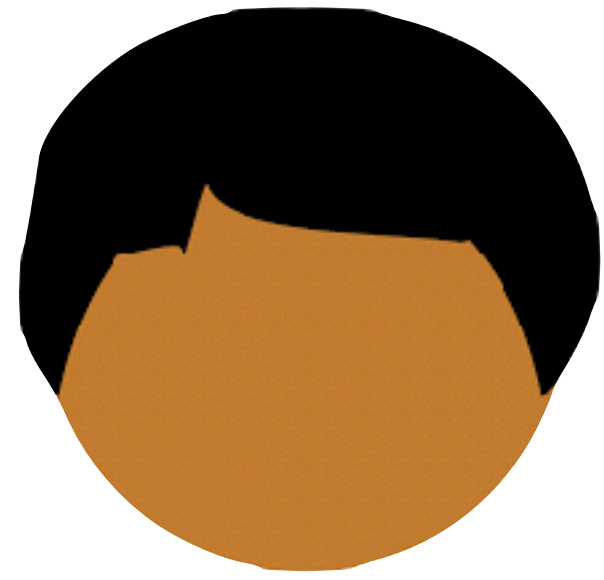
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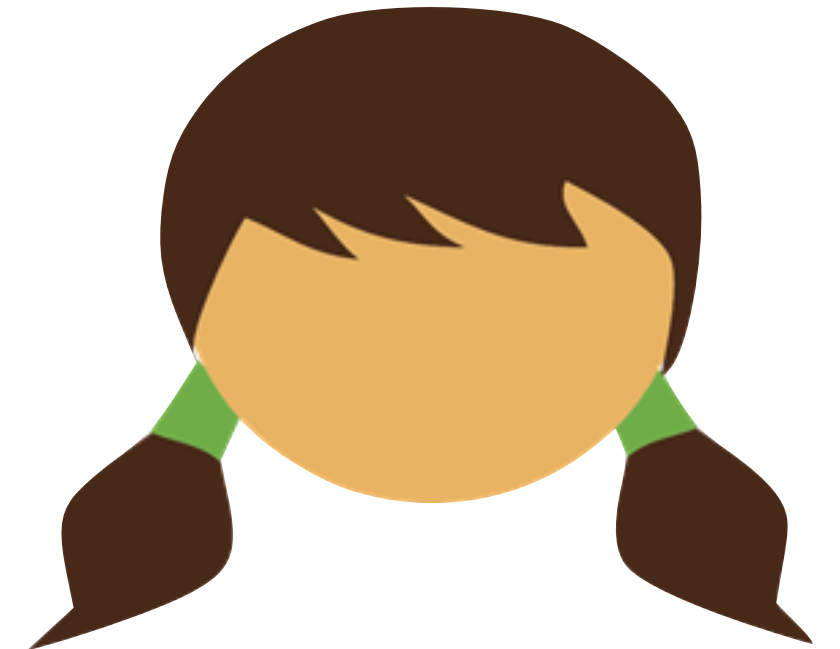
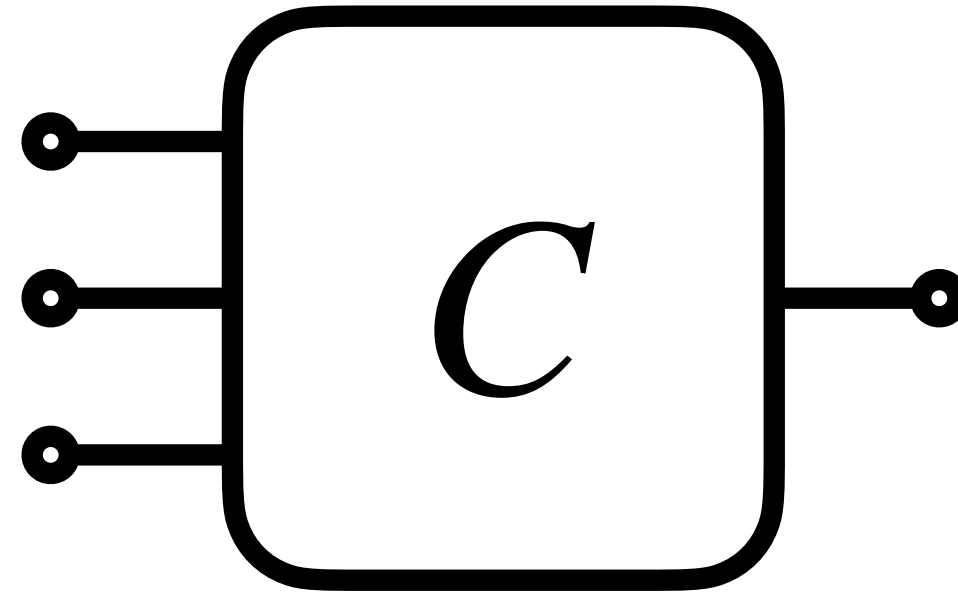
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Prover
Evaluator

Zero Knowledge from Garbled Circuits



Verifier
Garbler

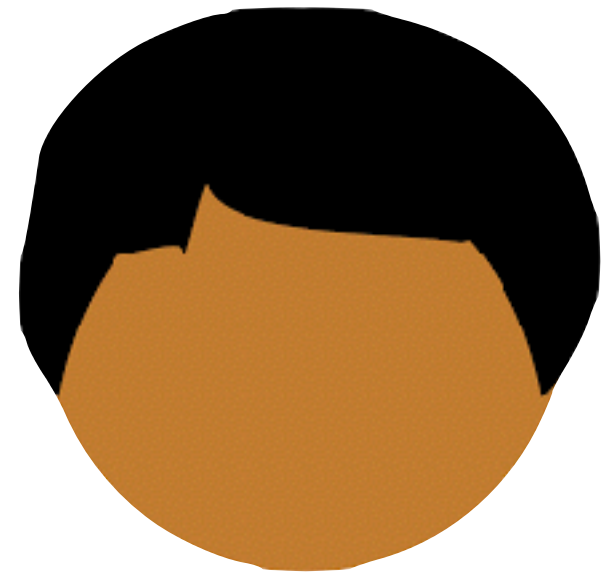


x

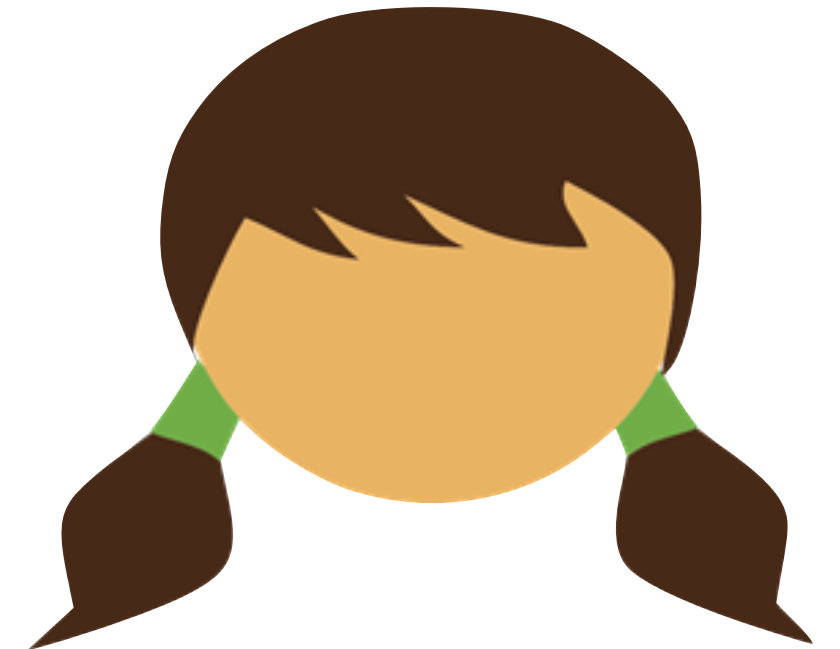
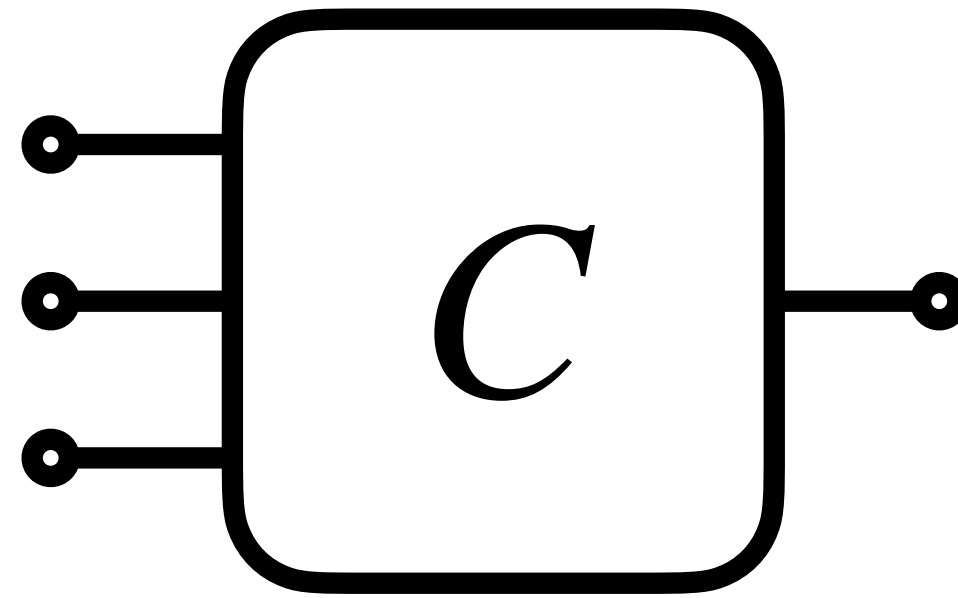
Prover
Evaluator

Intuition: Garbled Circuit provides natural protection against cheating evaluator

Zero Knowledge from Garbled Circuits



Verifier
Garbler

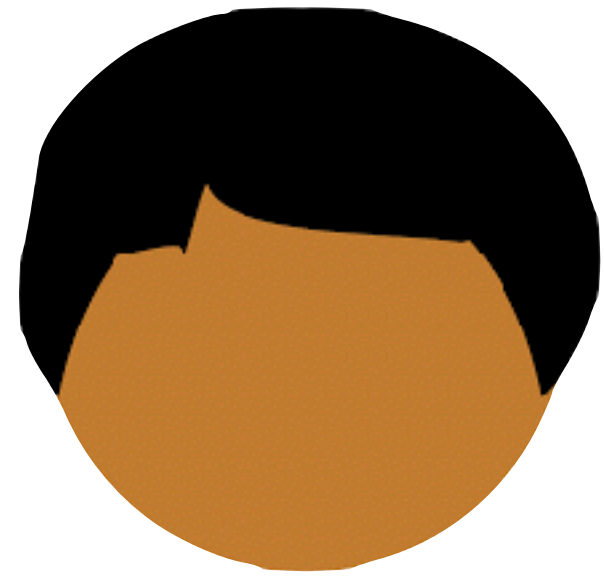


x
Prover
Evaluator

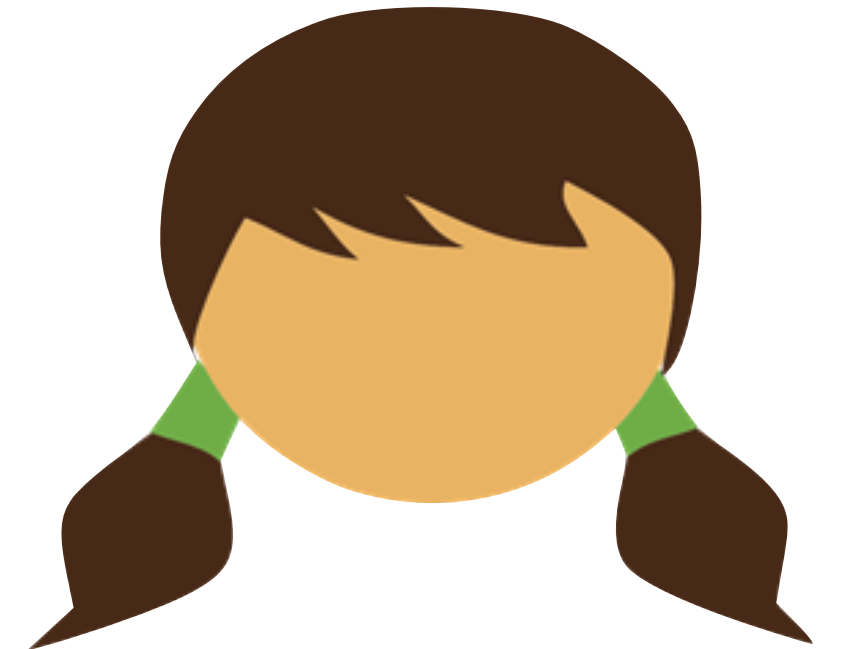
Intuition: Garbled Circuit provides natural protection against cheating evaluator

Just force evaluator to evaluate a garbling of C ; the fact that she can come up with an output key that encodes 1 is convincing evidence she has a witness

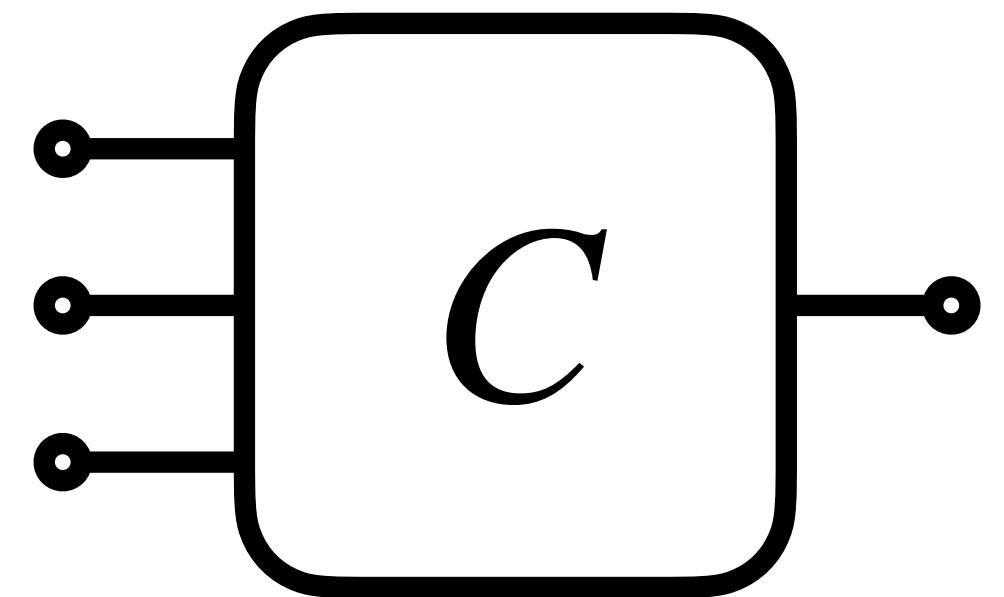
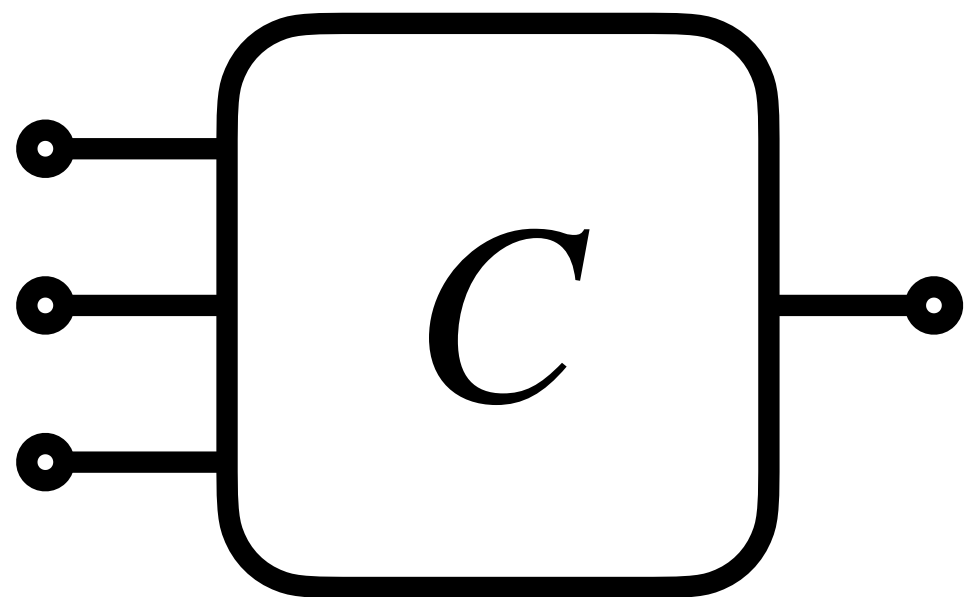
Zero Knowledge from Garbled Circuits



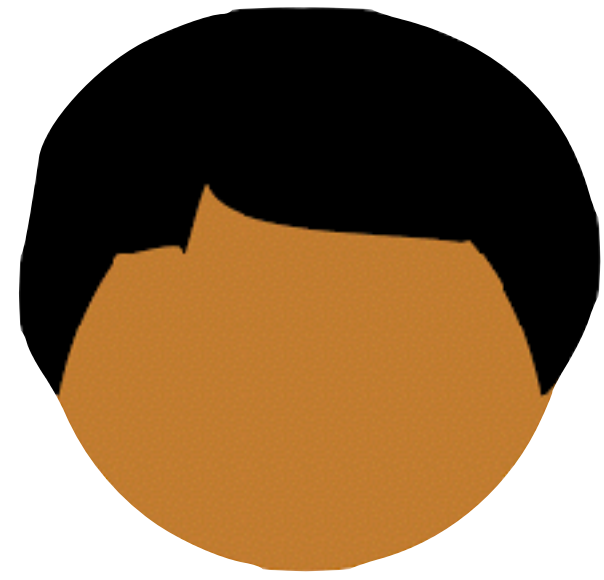
Verifier
Garbler



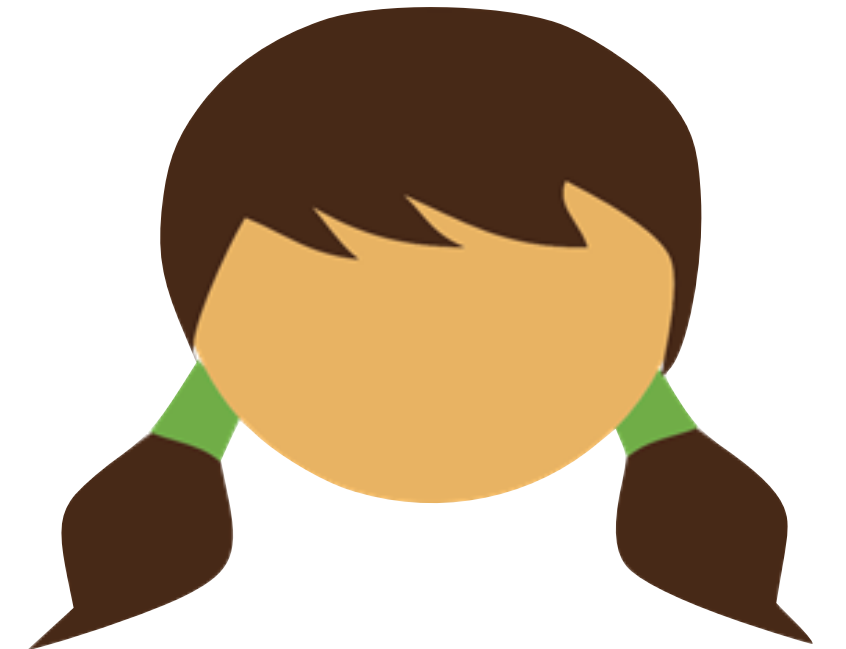
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Prover
Evaluator



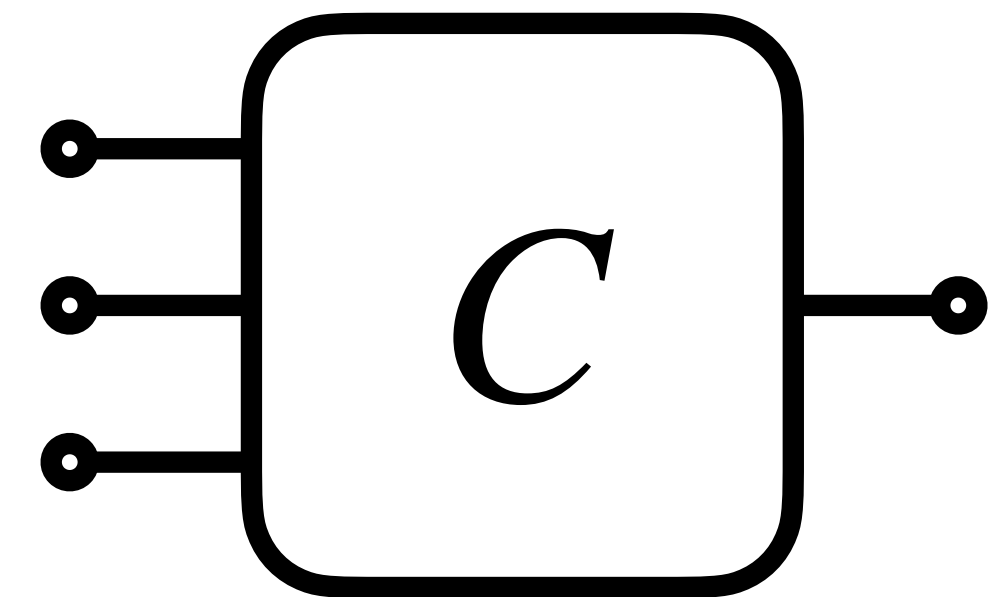
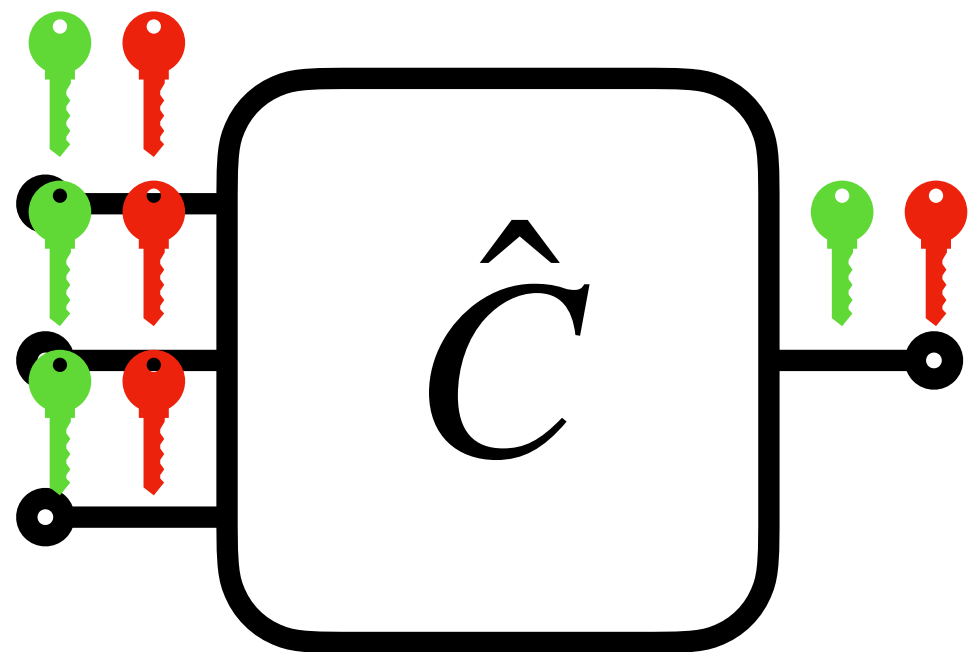
Zero Knowledge from Garbled Circuits



Verifier
Garbler



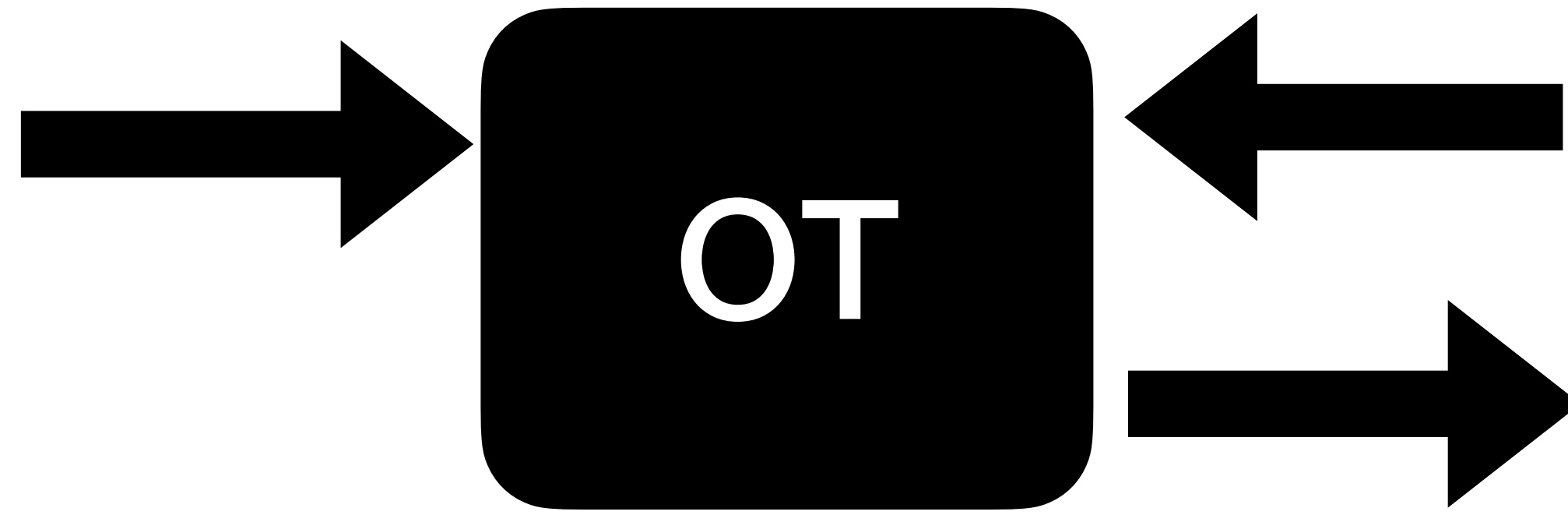
x
Prover
Evaluator



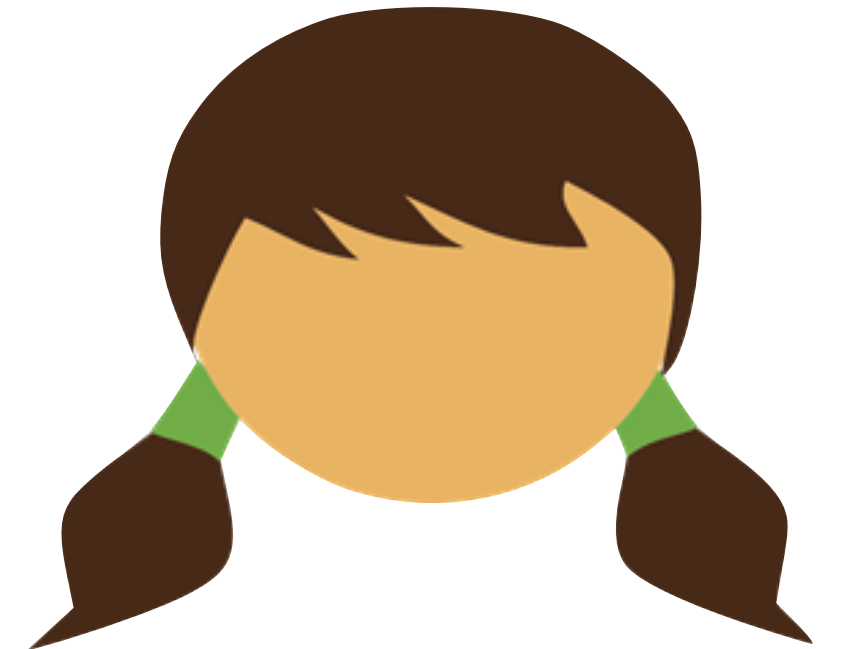
Zero Knowledge from Garbled Circuits



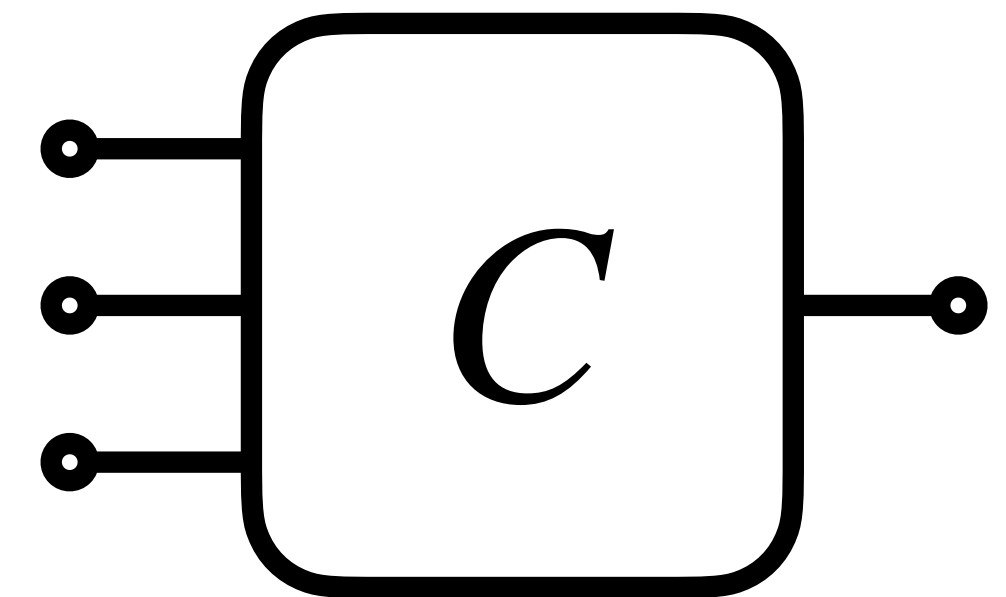
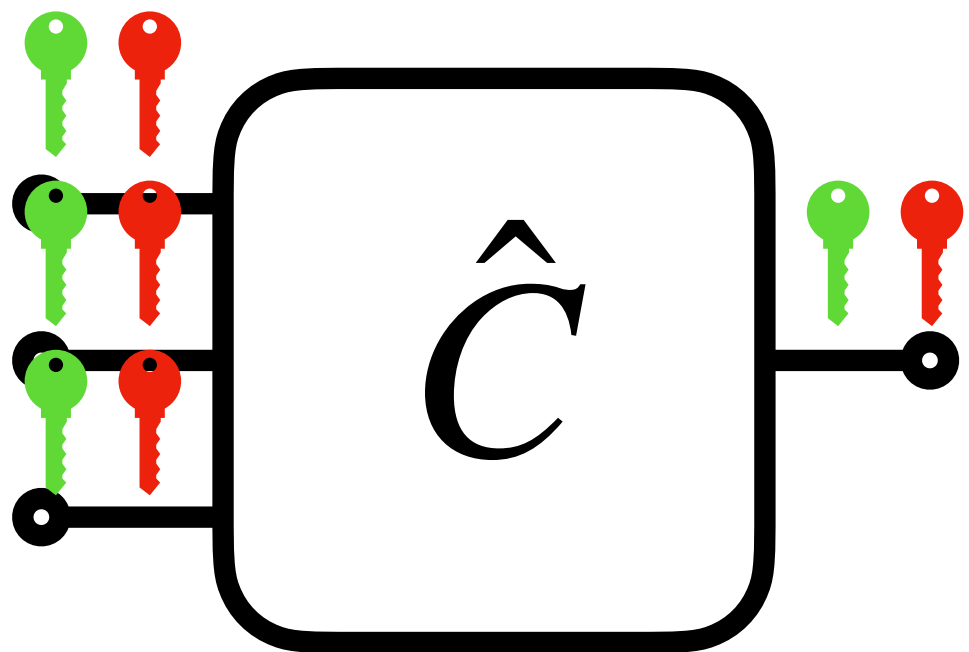
Verifier
Garbler



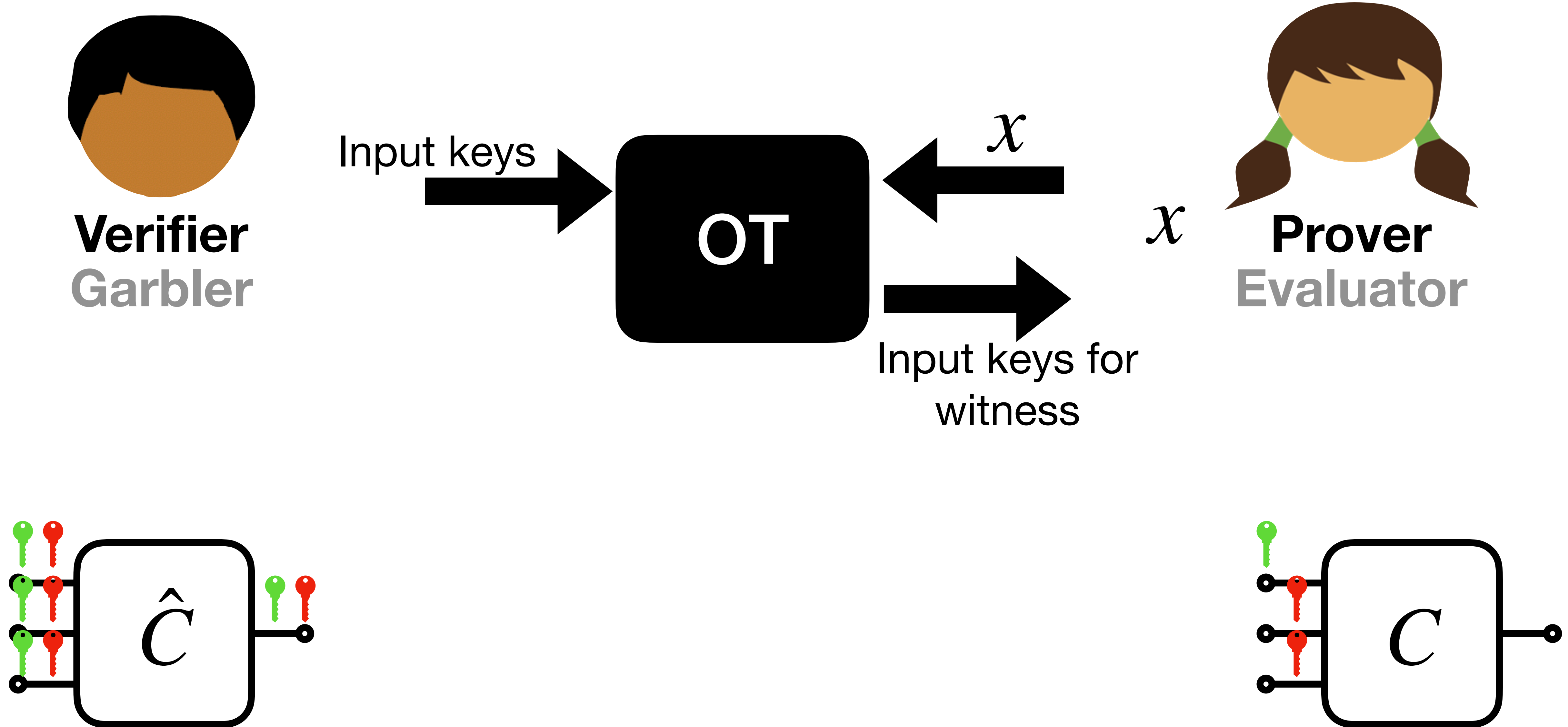
x



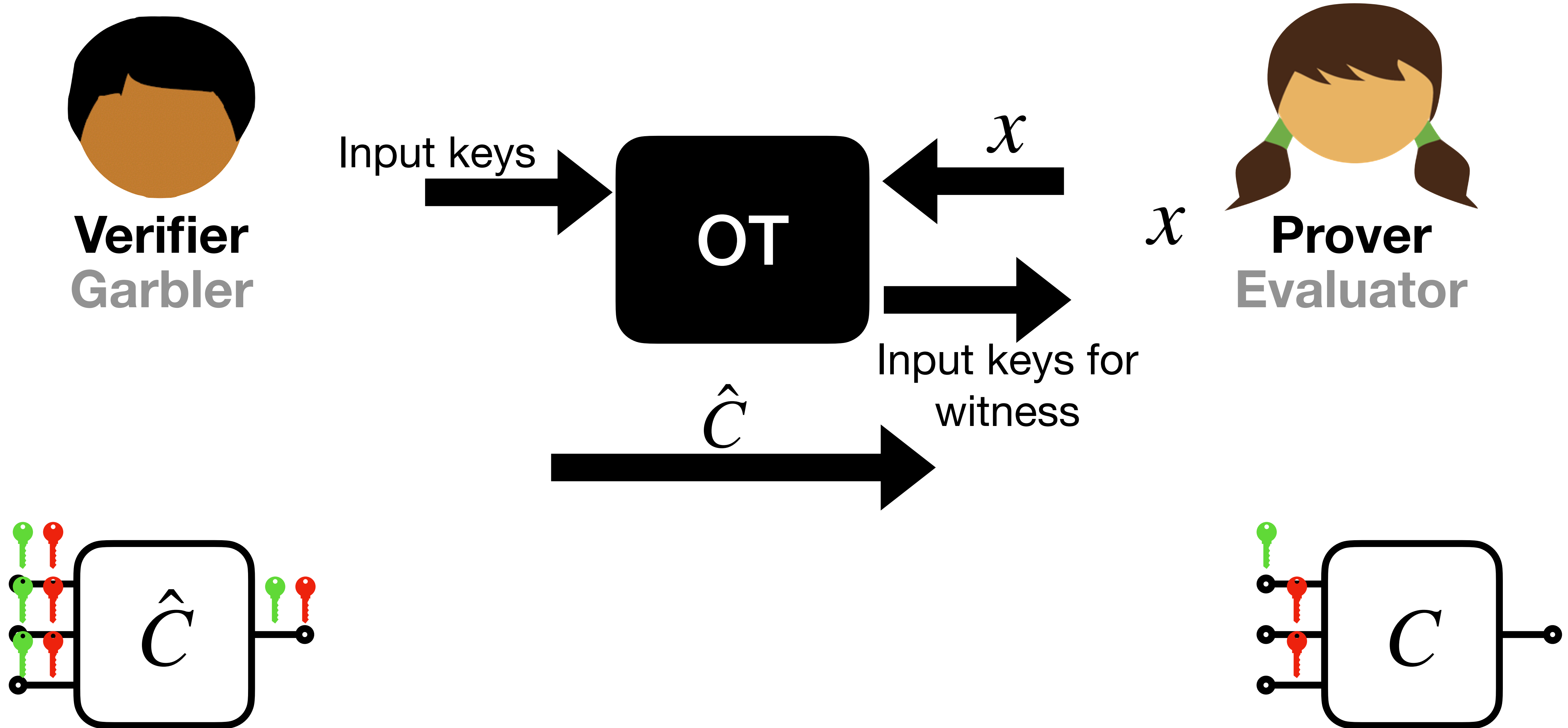
Prover
Evaluator



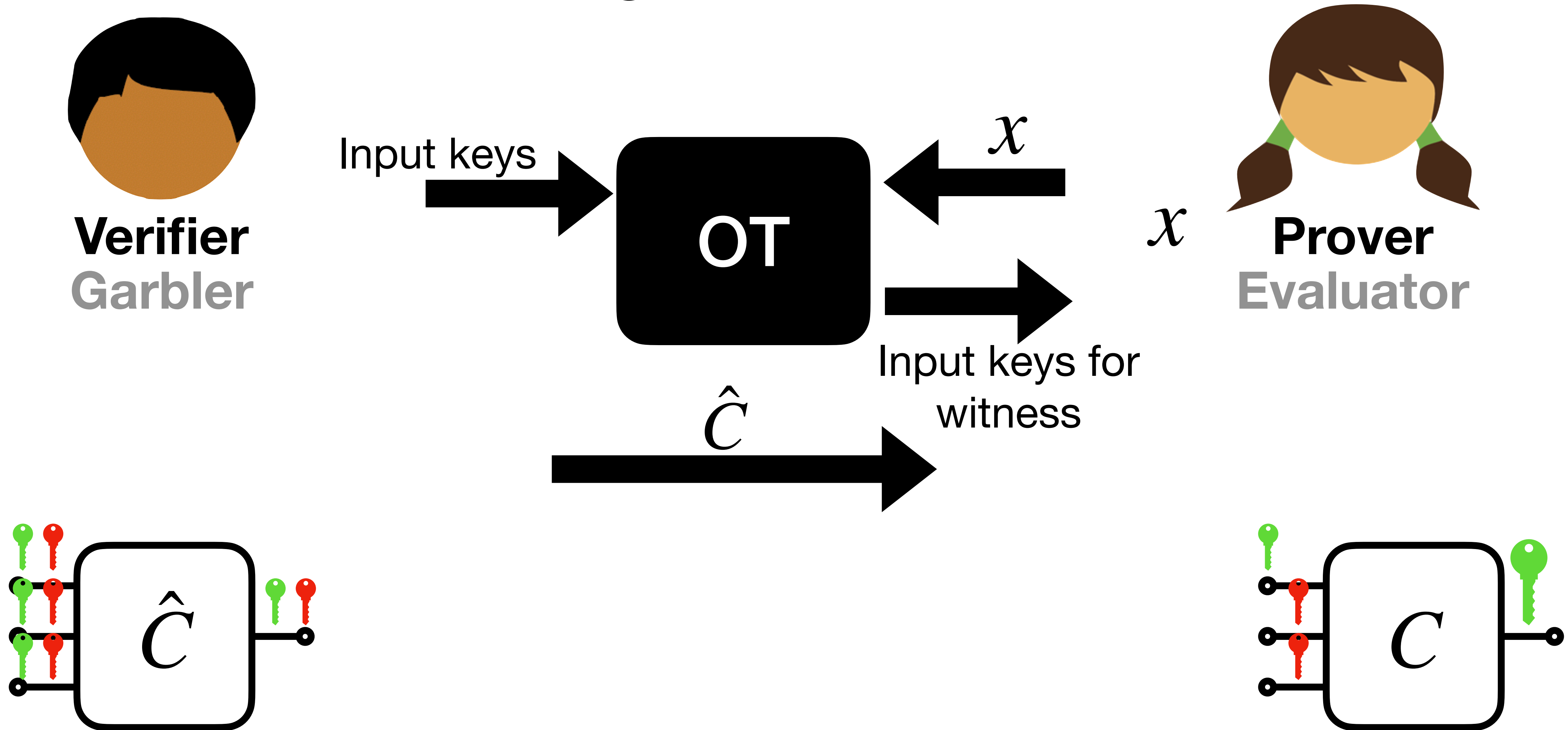
Zero Knowledge from Garbled Circuits



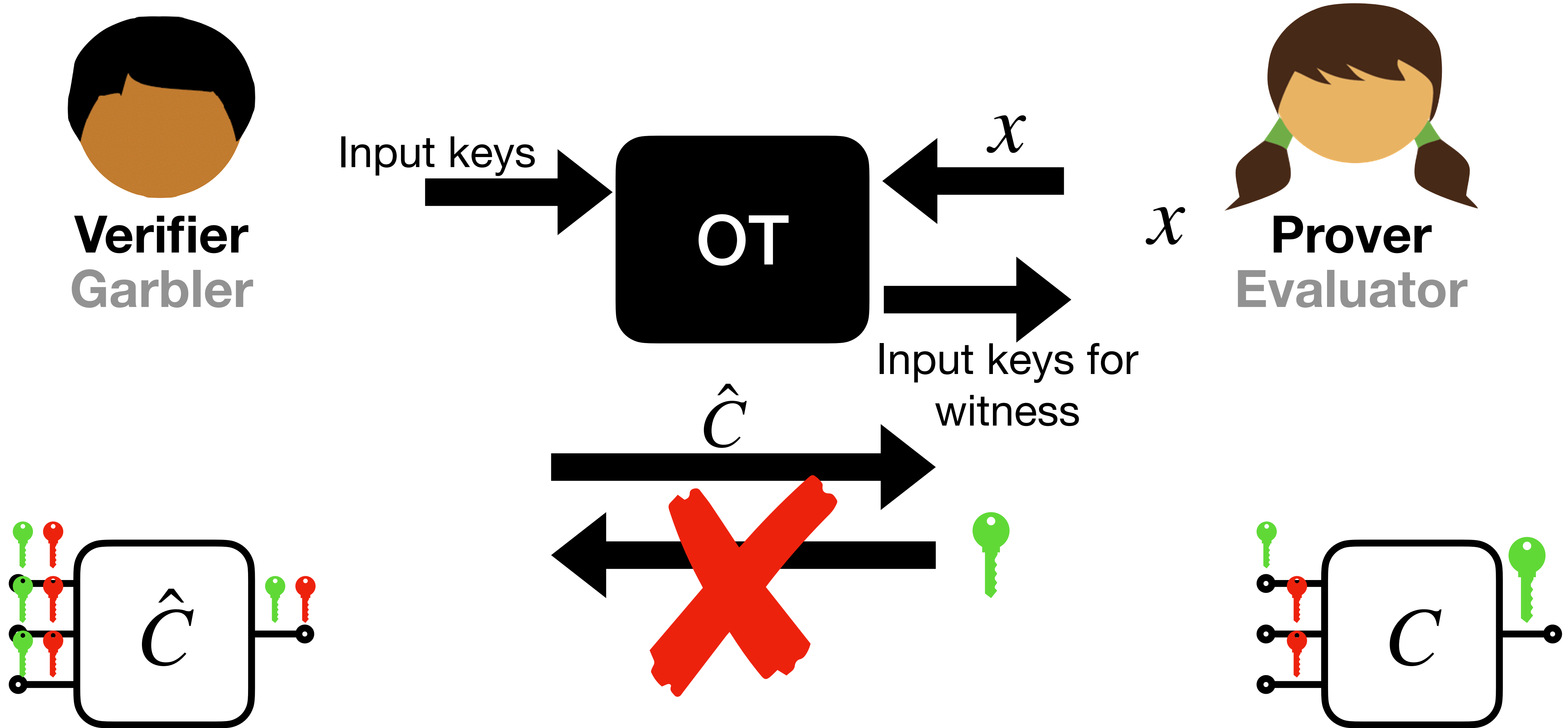
Zero Knowledge from Garbled Circuits



Zero Knowledge from Garbled Circuits



Zero Knowledge from Garbled Circuits





Verifier
Garbler

$$\text{Enc}(K_a^0, \text{Enc}(K_b^0, K_c^0))$$

$$\text{Enc}(K_a^0, \text{Enc}(K_b^1, K_c^0))$$

$$\text{Enc}(K_a^1, \text{Enc}(K_b^0, K_c^0))$$

$$\text{Enc}(K_a^1, \text{Enc}(K_b^1, K_c^1))$$



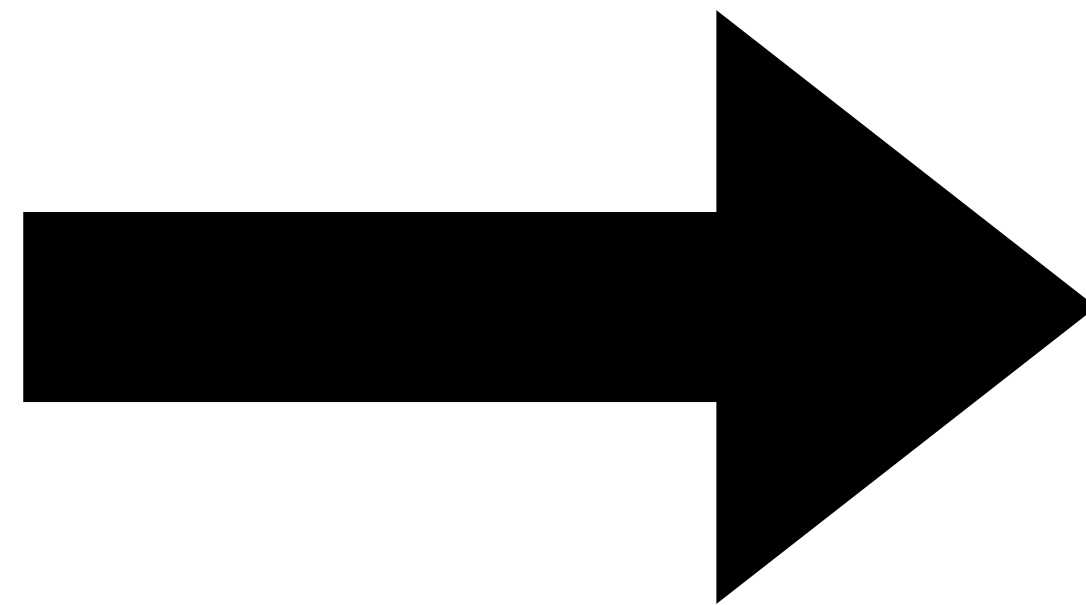
Verifier
Garbler

$$\text{Enc}(K_a^0, \text{Enc}(K_b^0, K_c^0))$$

$$\text{Enc}(K_a^0, \text{Enc}(K_b^1, K_c^0))$$

$$\text{Enc}(K_a^1, \text{Enc}(K_b^0, K_c^0))$$

$$\text{Enc}(K_a^1, \text{Enc}(K_b^1, K_c^1))$$



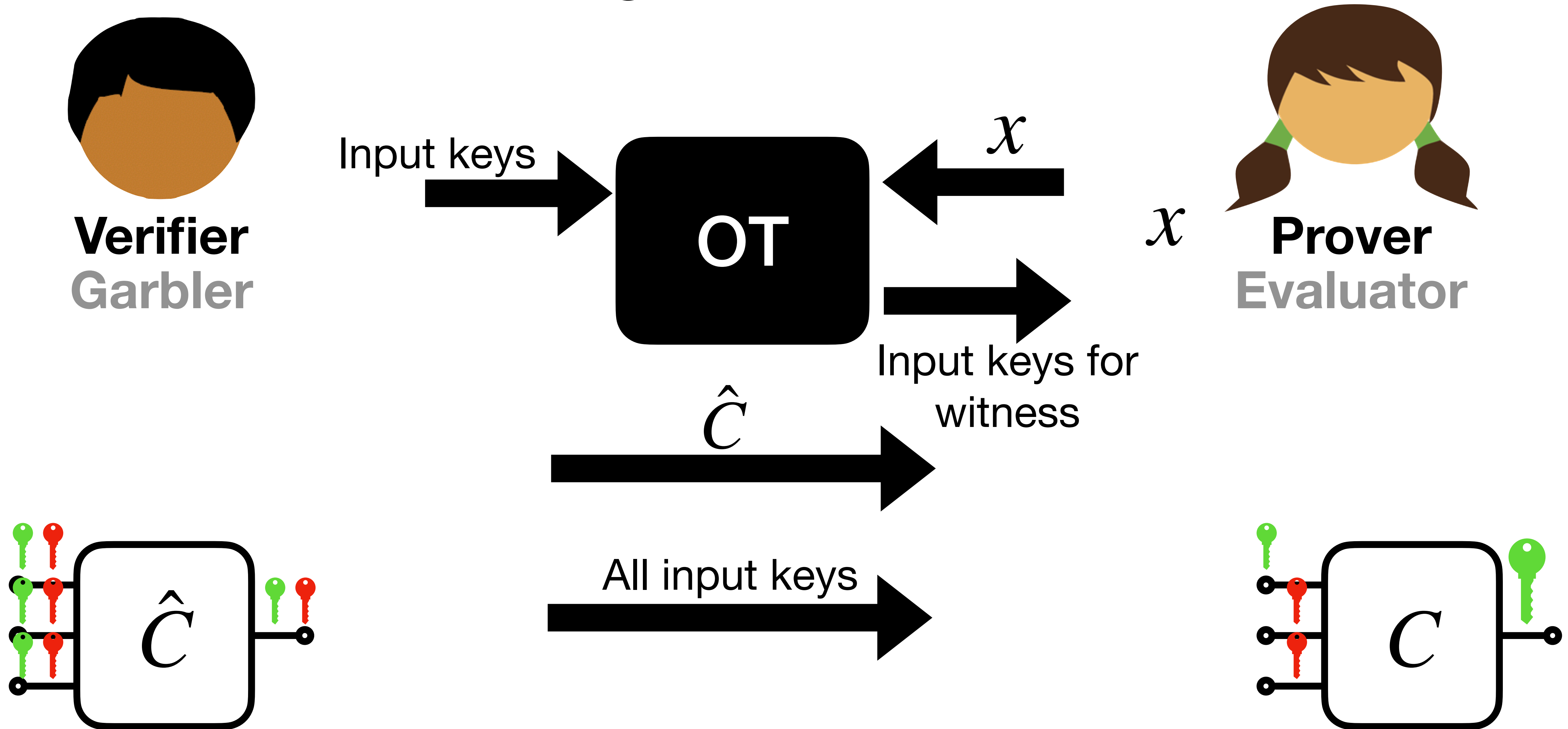
$$\text{Enc}(K_a^0, \text{Enc}(K_b^0, K_c^1))$$

$$\text{Enc}(K_a^0, \text{Enc}(K_b^1, K_c^1))$$

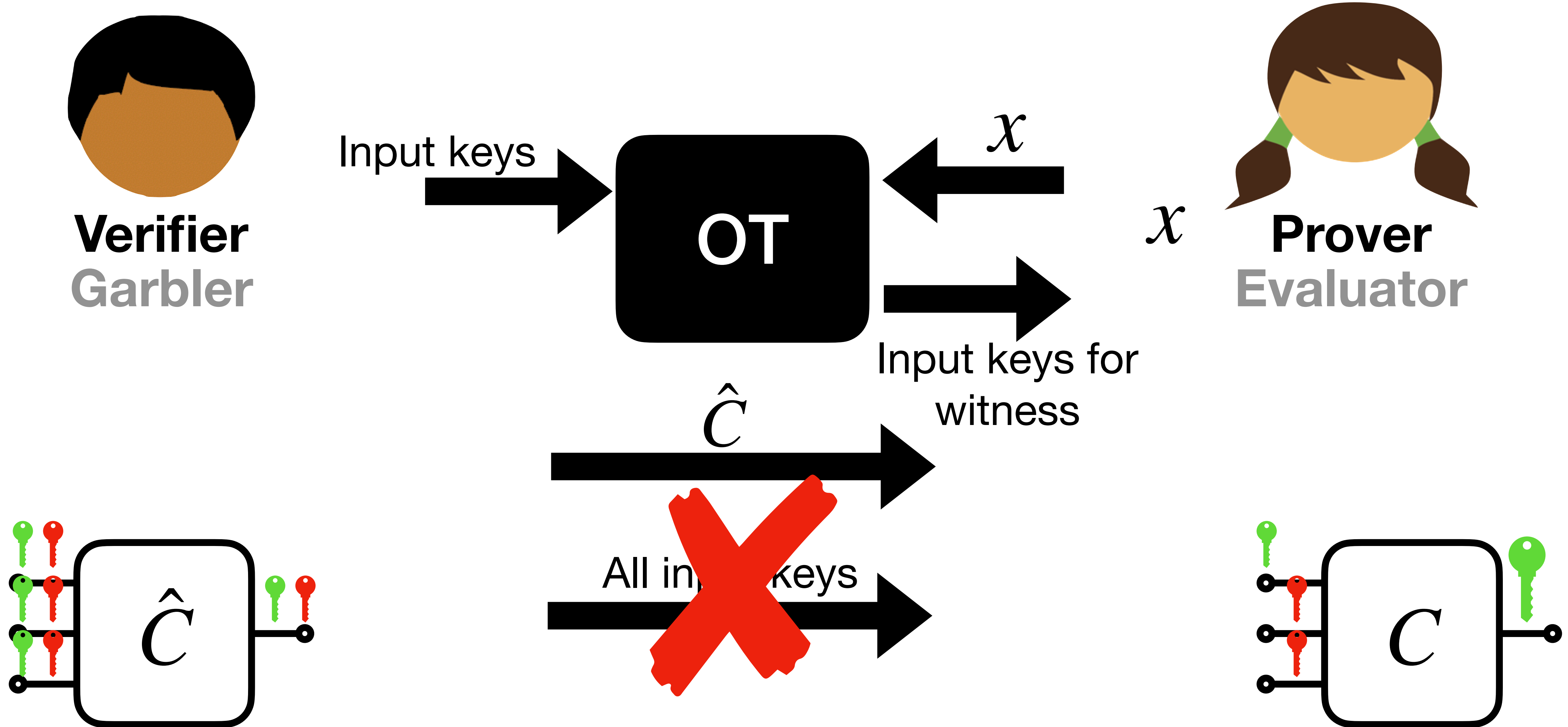
$$\text{Enc}(K_a^1, \text{Enc}(K_b^0, K_c^1))$$

$$\text{Enc}(K_a^1, \text{Enc}(K_b^1, K_c^1))$$

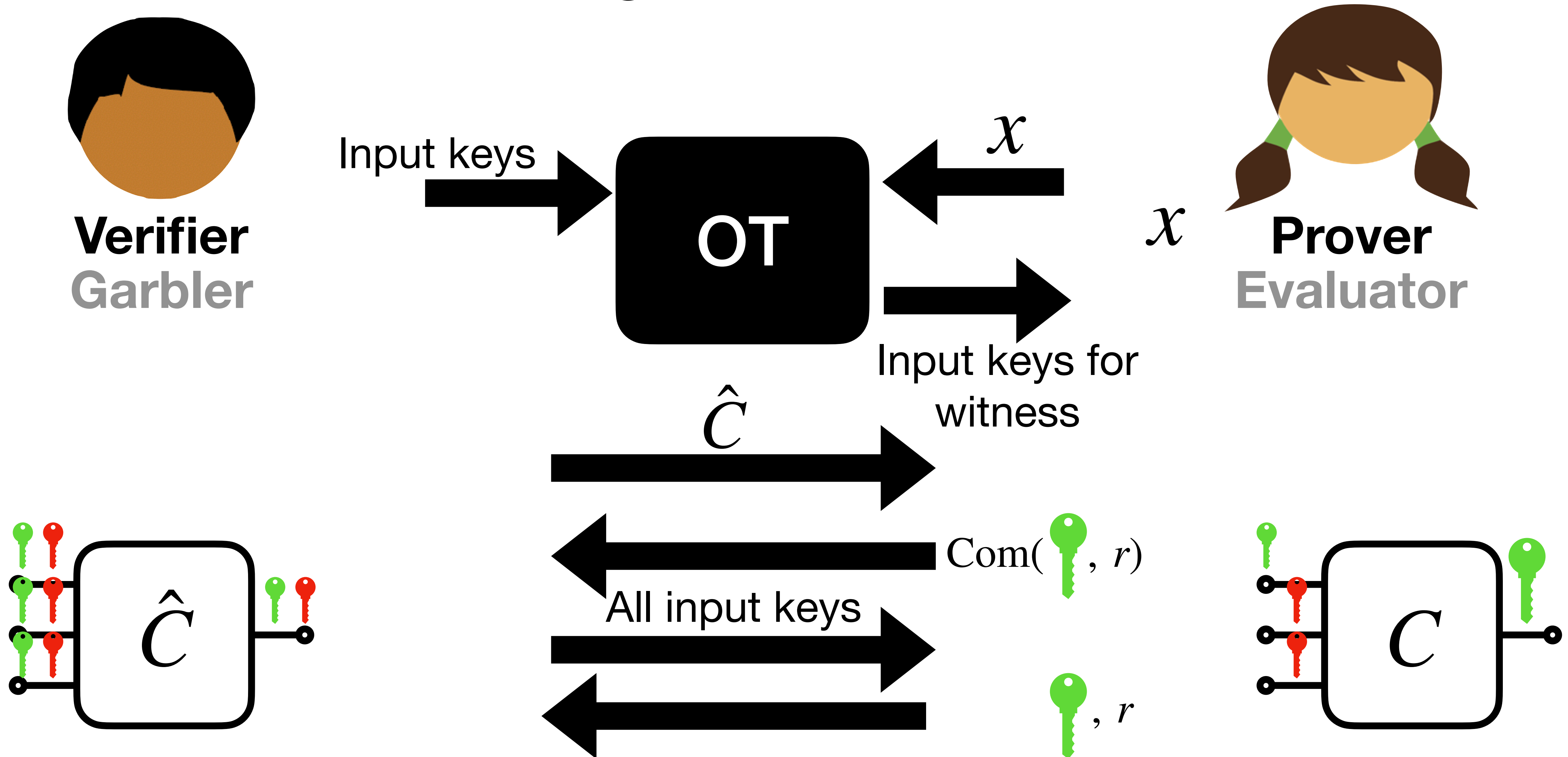
Zero Knowledge from Garbled Circuits



Zero Knowledge from Garbled Circuits



Zero Knowledge from Garbled Circuits



Zero Knowledge from Garbled Circuits

Completeness? (Honest prover can prove true things)

Zero Knowledge from Garbled Circuits

Completeness? (Honest prover can prove true things)

From **correctness** of GC

Zero Knowledge from Garbled Circuits

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Soundness? (Malicious prover cannot prove false things)

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Zero Knowledge from Garbled Circuits

Completeness? (Honest prover can prove true things)

From **correctness** of GC

Soundness? (Malicious prover cannot prove false things)

From **authenticity** of GC

Zero Knowledge? (Malicious verifier learns nothing)

Verifier is forced to produce a valid GC, and sees only the encoding of a one output key

Simulator can get all input keys from verifier, then send the one key to the verifier

Zero-Knowledge from Secure Multiparty Computation*

Yuval Ishai[†] Eyal Kushilevitz[‡] Rafail Ostrovsky[§] Amit Sahai[¶]

Abstract

A *zero-knowledge proof* allows a prover to convince a verifier of an assertion without revealing any further information beyond the fact that the assertion is true. *Secure multiparty computation* allows n mutually suspicious players to jointly compute a function of their local inputs without revealing to any t corrupted players additional information beyond the output of the function.

We present a new general connection between these two fundamental notions. Specifically, we present a general construction of a zero-knowledge proof for an NP relation $R(x, w)$ which only makes a *black-box* use of any secure protocol for a related *multi-party* functionality f . The latter protocol is only required to be secure against a small number of “honest but curious” players. We also present a variant of the basic construction that can leverage security against a large number of *malicious* players to obtain better efficiency.

As an application, one can translate previous results on the efficiency of secure multiparty computation to the domain of zero-knowledge, improving over previous constructions of efficient zero-knowledge proofs. In particular, if verifying R on a witness of length m can be done by a circuit C of size s , and assuming one-way functions exist, we get the following types of zero-knowledge proof protocols:

- **Approaching the witness length.** If C has constant depth over $\wedge, \vee, \oplus, \neg$ gates of unbounded fan-in, we get a zero-knowledge proof protocol with communication complexity $m \cdot \text{poly}(k) \cdot \text{polylog}(s)$, where k is a security parameter.
- **“Constant-rate” zero-knowledge.** For an *arbitrary* circuit C of size s and a bounded fan-in, we get a zero-knowledge protocol with communication complexity $O(s) + \text{poly}(k, \log s)$. Thus, for large circuits, the ratio between the communication complexity and the circuit size approaches a constant. This improves over the $O(ks)$ complexity of the best previous protocols.

Keywords: Cryptography, zero-knowledge, secure computation, black-box reductions

*A preliminary version of this paper appeared in STOC 2007 [32]. Work done in part while the authors were visiting IPAM.
[†]Computer Science Department, Technion and UCLA. Email: yuvali@cs.technion.ac.il. Supported by BSF grant 2004361, ISF grant 1310/06, and NSF grants 0205594, 0430254, 0456717, 0627781, 0716835, 0716389.
[‡]Computer Science Department, Technion. Email: eyalk@cs.technion.ac.il. Research supported by grant 1310/06 from the Israel Science Foundation and by grant 2002354 from the U.S.-Israel Binational Science Foundation.
[§]Computer Science Department and Department of Mathematics, UCLA. Email: rafail@cs.ucla.edu. Supported in part by IBM Faculty Award, Xerox Innovation Group Award, NSF grants 0430254, 0716835, 0716389, 0830803 and U.C. MICRO grant.
[¶]Computer Science Department, UCLA. Email: sahai@cs.ucla.edu. Research supported in part from grants from the NSF ITR and Cybertrust programs (including grants 0627781, 0456717, 0830803, and 0205594), BSF grant 2004361, a subgrant from SRI as part of the Army Cyber-TA program, an equipment grant from Intel, an Alfred P. Sloan Foundation Fellowship, and an Okawa Foundation Research Grant.

MPC in the Head Paradigm

Non-interactive Zero Knowledge

Can give succinct proofs

Plausibly post-quantum secure

“Meta”-quality of some kinds of cryptography

INVITED AND ACCEPTED TO SIAM JOURNAL ON COMPUTING (SICOMP) SPECIAL ISSUE DEVOTED TO STOC-2007.

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ZKBoo: Faster Zero-Knowledge for Boolean Circuits

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Post-Quantum Zero-Knowledge and Signatures from Symmetric-Key Primitives*

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Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures

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Session J: Outsourcing

CCS'17, October 30–November 3, 2017, Dallas, TX, USA

Ligero: Lightweight Sublinear Arguments Without a Trusted Setup

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ABSTRACT

We design and implement a simple zero-knowledge argument protocol for NP whose communication complexity is proportional to the square-root of the verification circuit size. The protocol can be based on any collision-resistant hash function. Alternatively, it can be made non-interactive in the random oracle model, yielding concretely efficient zk-SNARKs that do not require a trusted setup or public-key cryptography.

Our protocol is attractive not only for very large verification circuits but also for moderately large circuits that arise in appli-

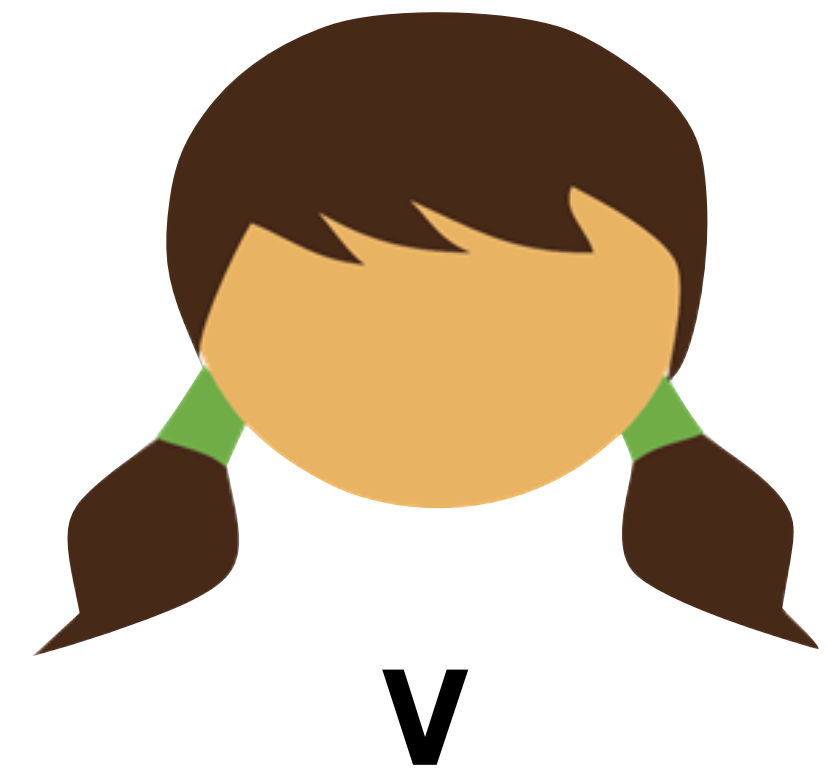
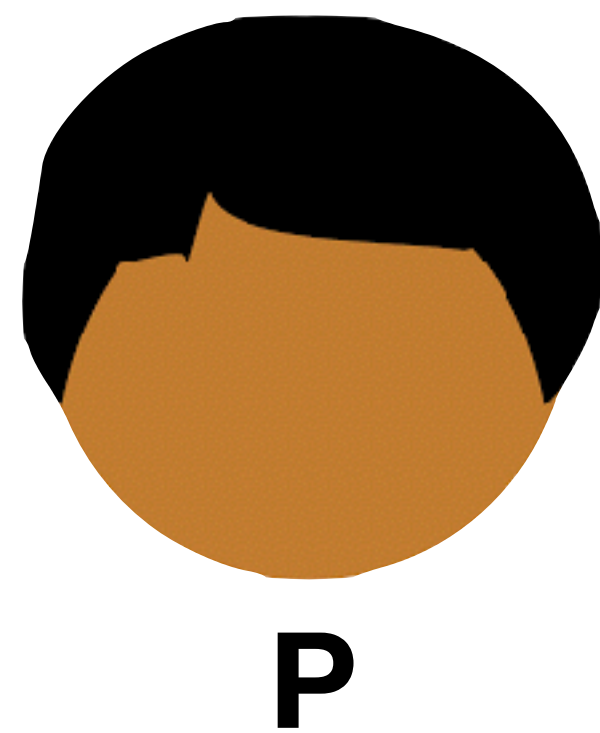
a concretely efficient argument protocol for NP whose communication complexity is proportional to the square root of the size of a circuit verifying the NP witness. Our argument system is in fact a zero-knowledge argument of knowledge, and it only requires the verifier to send public coins to the prover. The latter feature implies that it can be made non-interactive via the Fiat-Shamir transform [19], yielding an efficient implementation of zero-knowledge succinct non-interactive arguments of knowledge (zk-SNARKs [11]) without a trusted setup.

To put our work in the proper context, we give some relevant background. The best of knowledge-based arguments have been in

ZK from MPC in the Head

Statements: Boolean circuit satisfiability

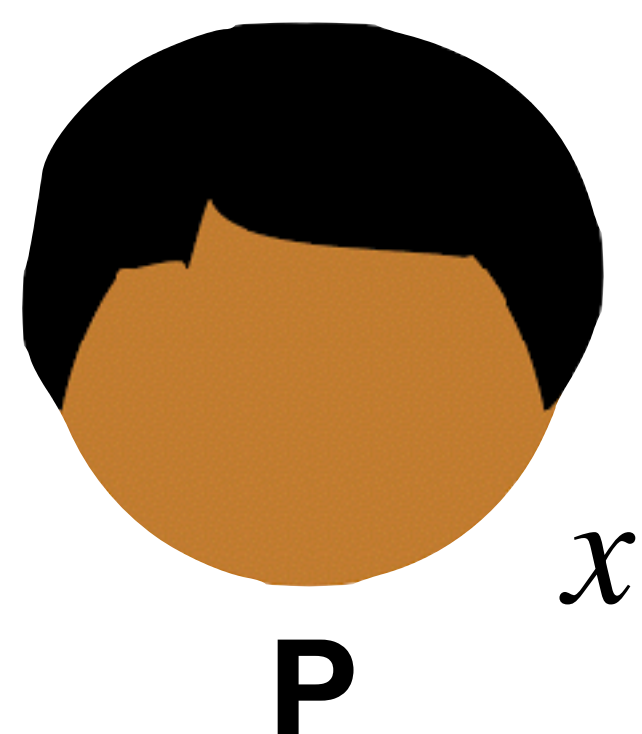
Witness: circuit input causing circuit to output 1



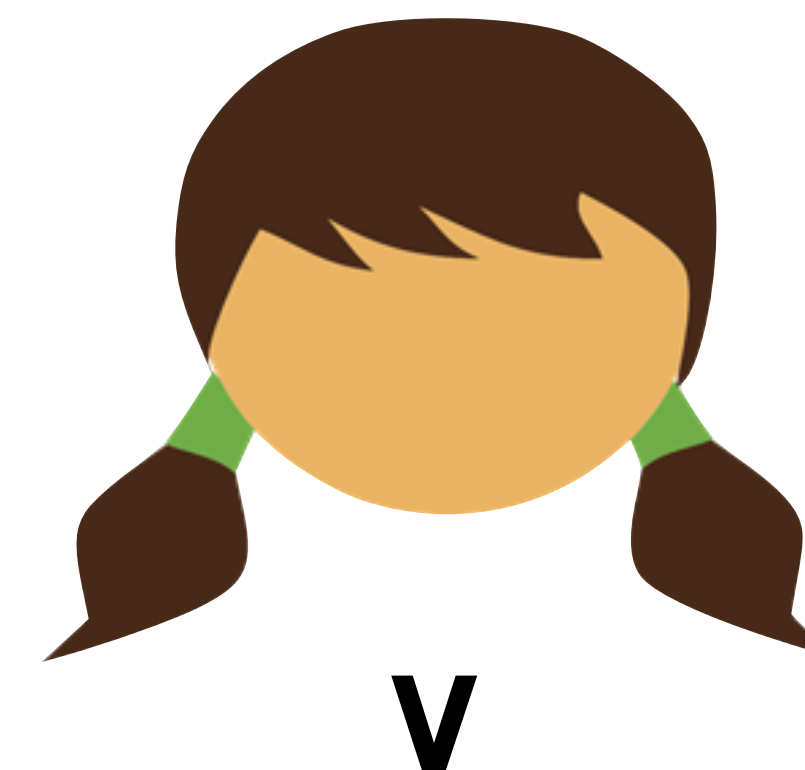
ZK from MPC in the Head

Statements: Boolean circuit satisfiability

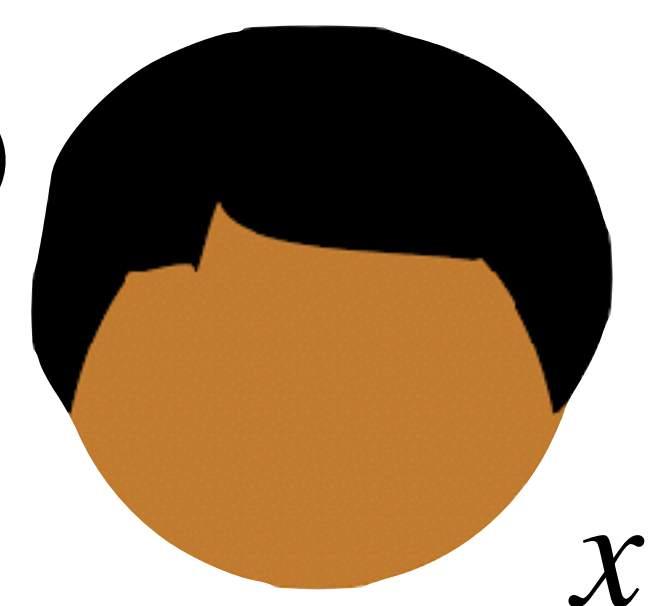
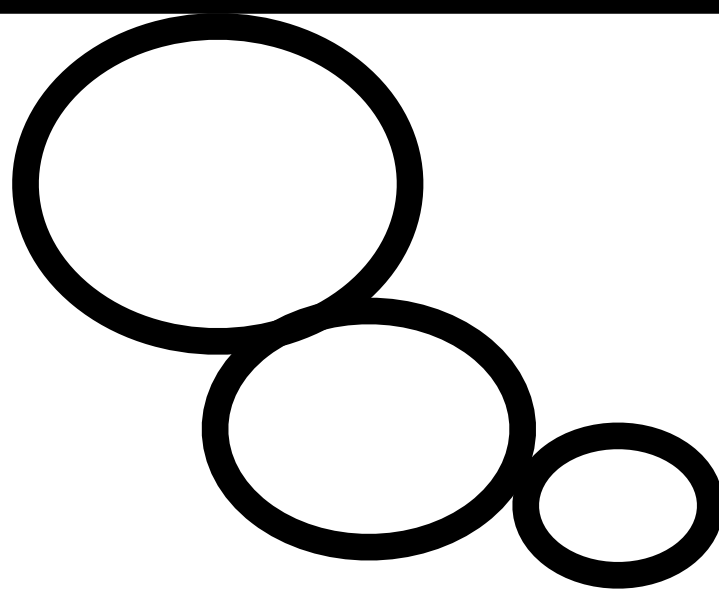
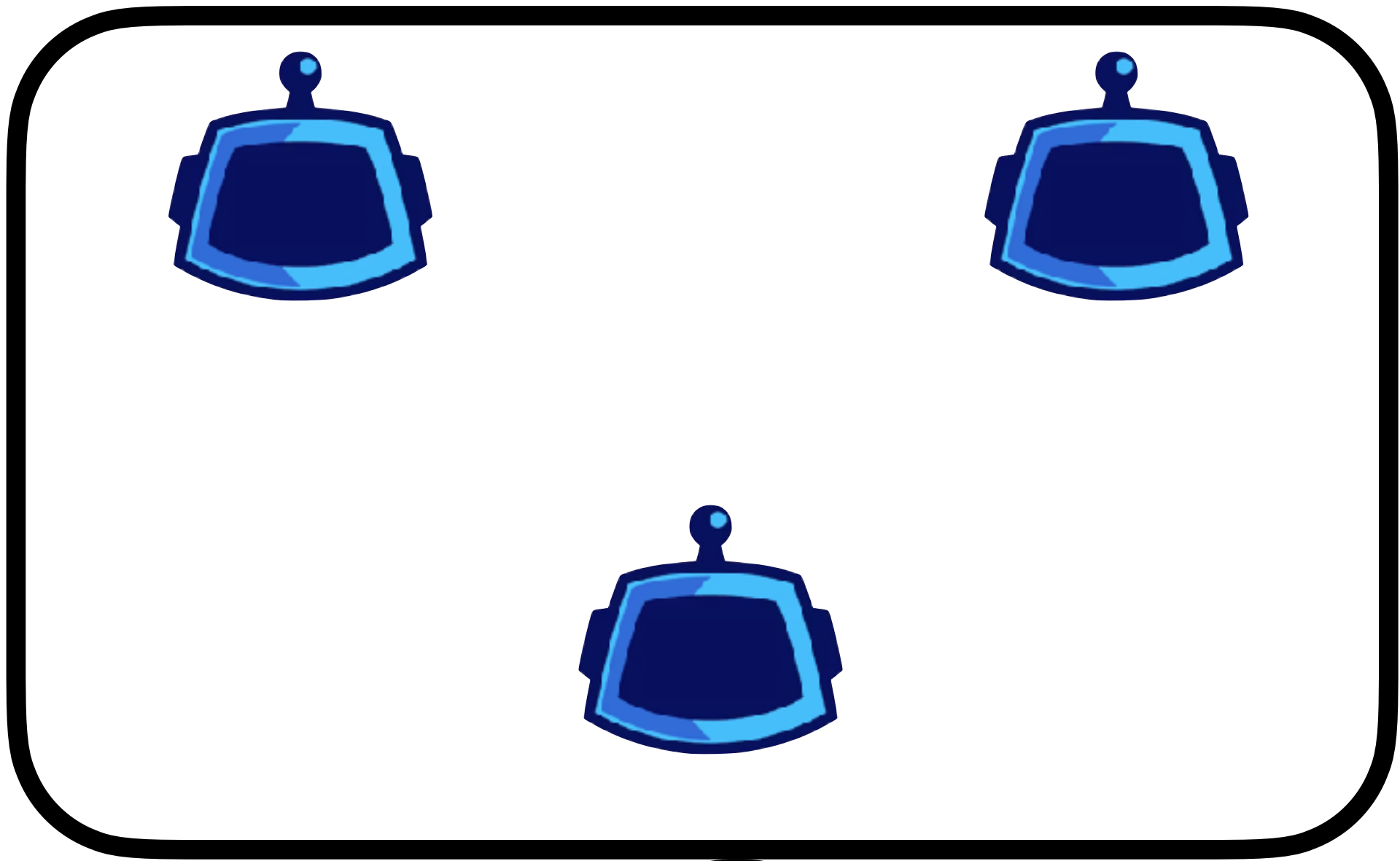
Witness: circuit input causing circuit to output 1



C



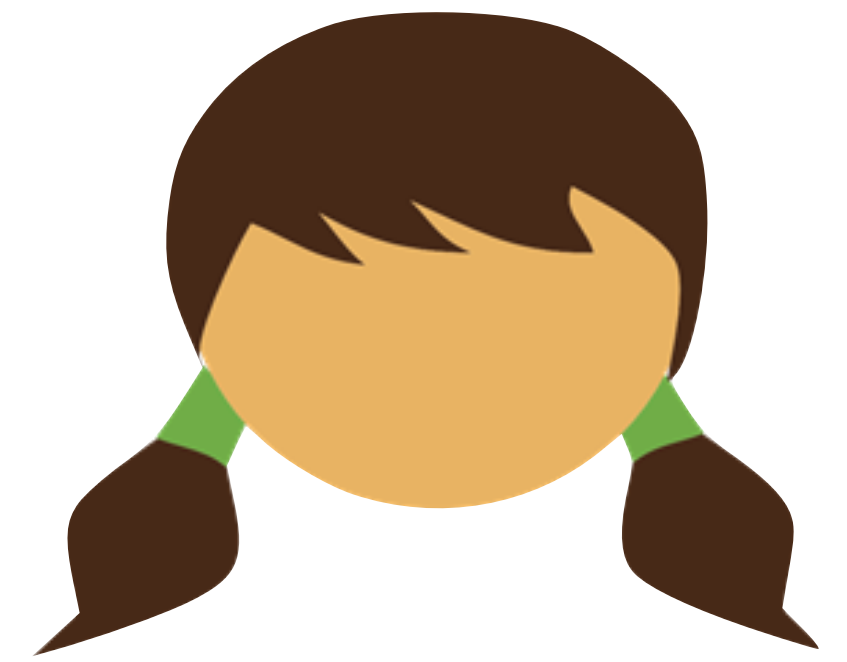
ZK from MPC in the Head



P

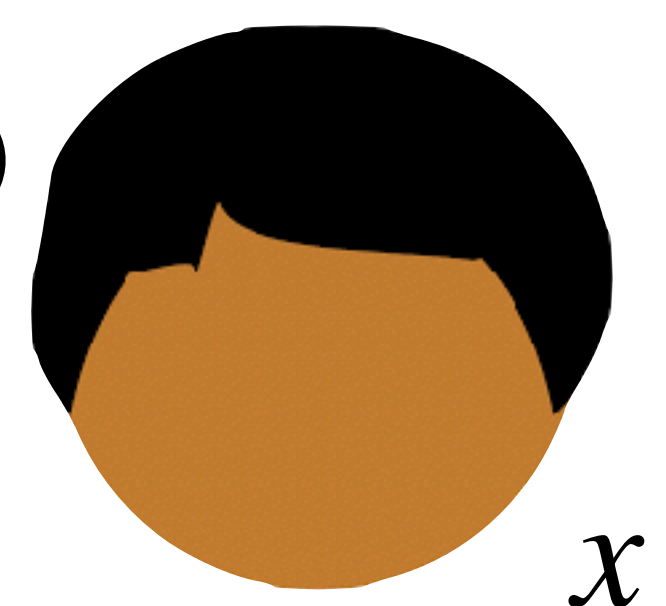
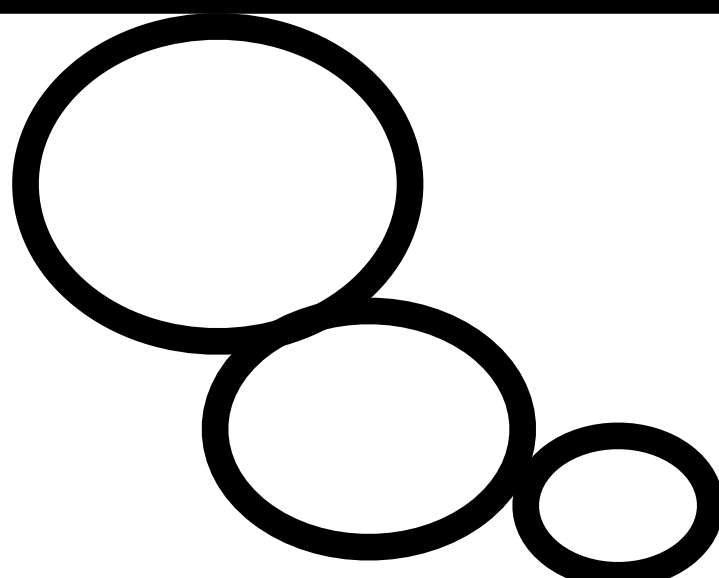
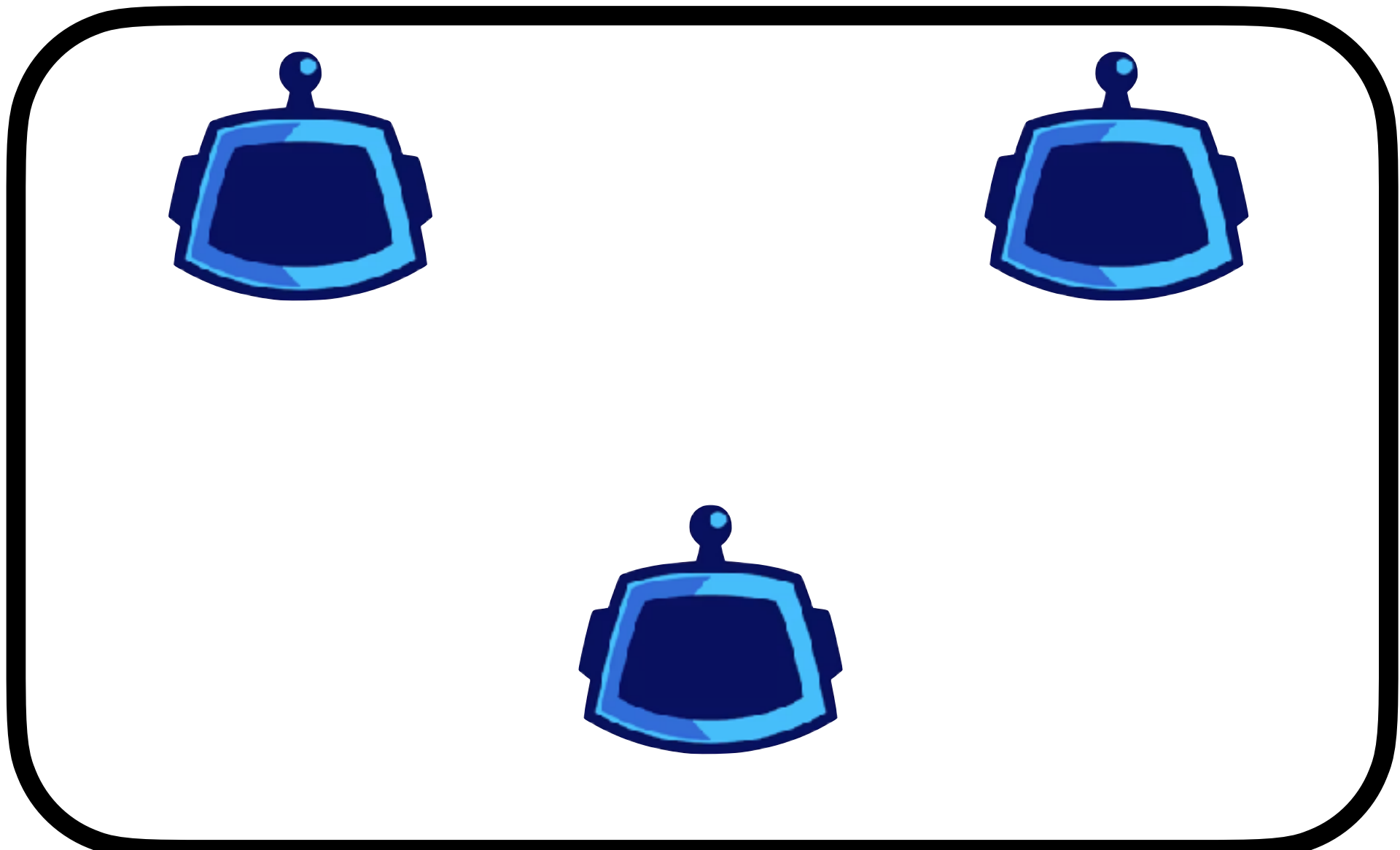
x

C



V

ZK from MPC in the Head



P

x

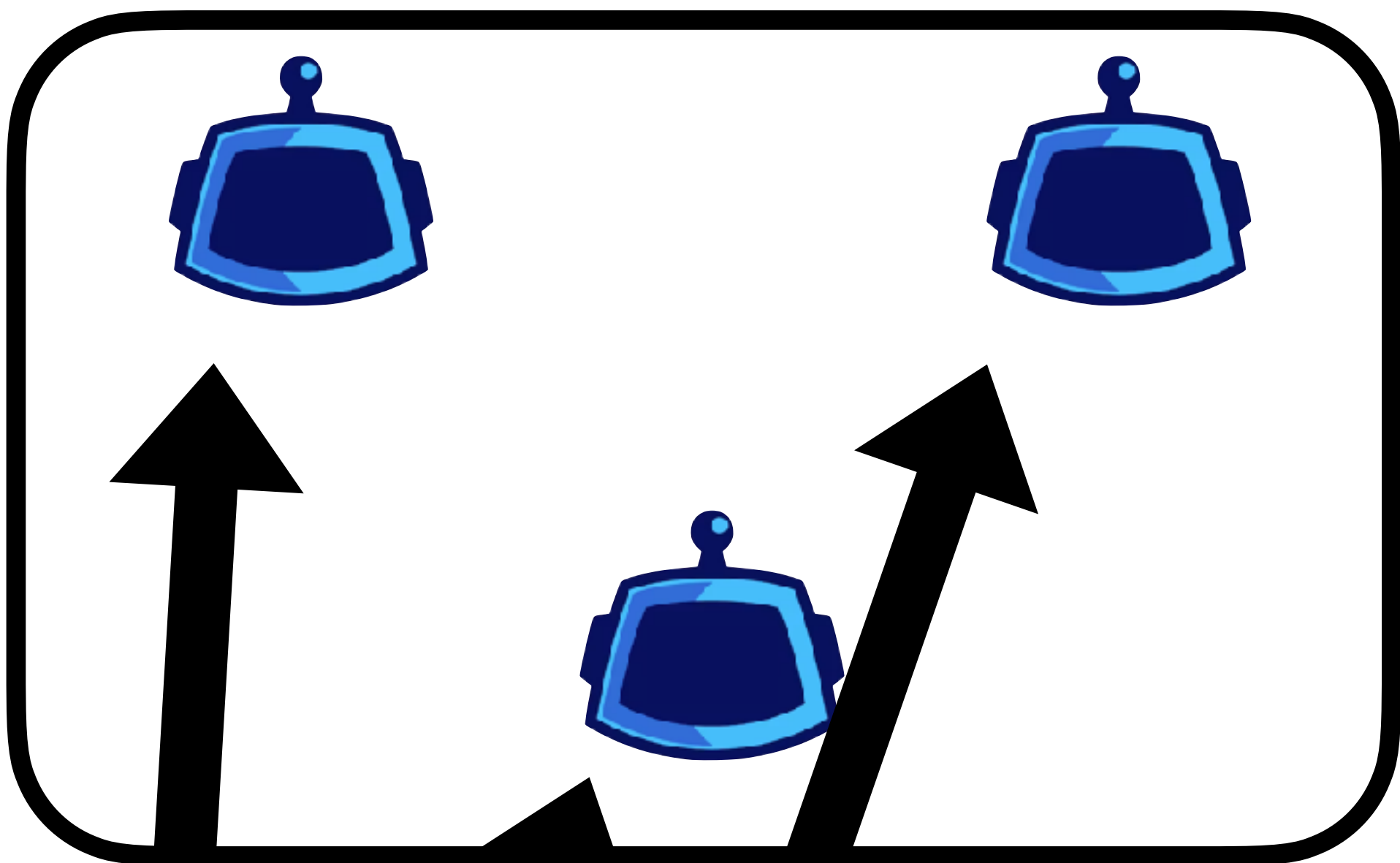
$x_0 \oplus x_1 \oplus x_2 = x$
secret shares

C

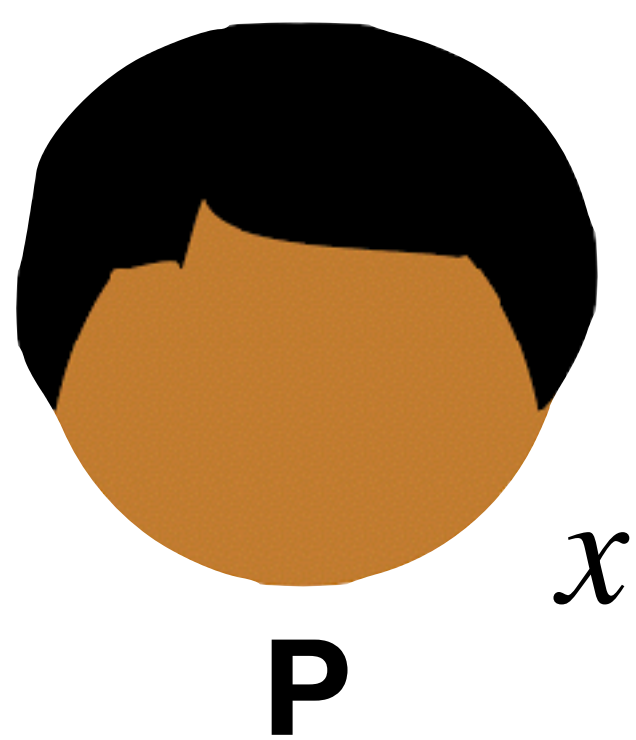


V

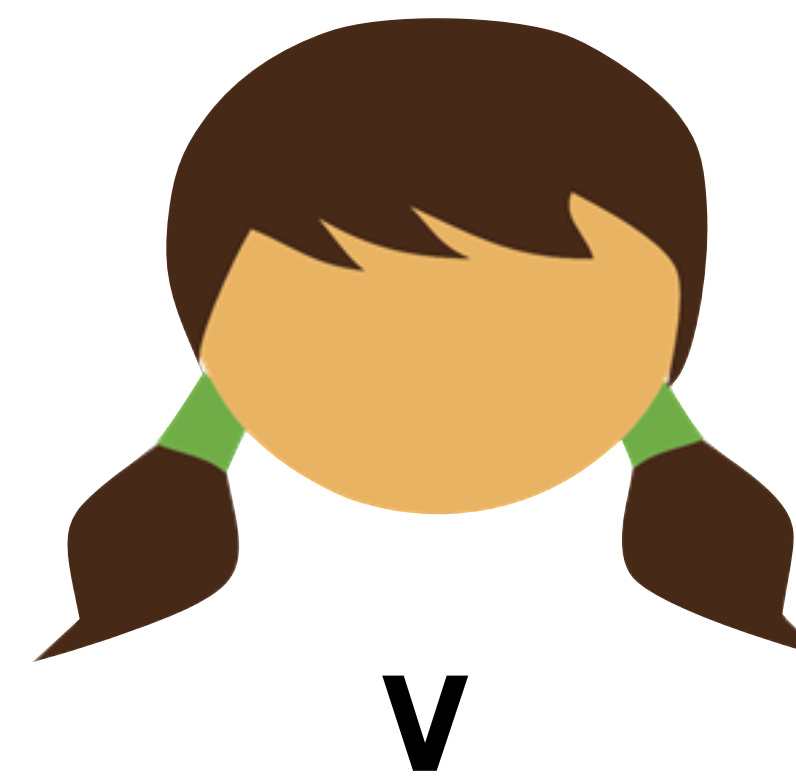
ZK from MPC in the Head



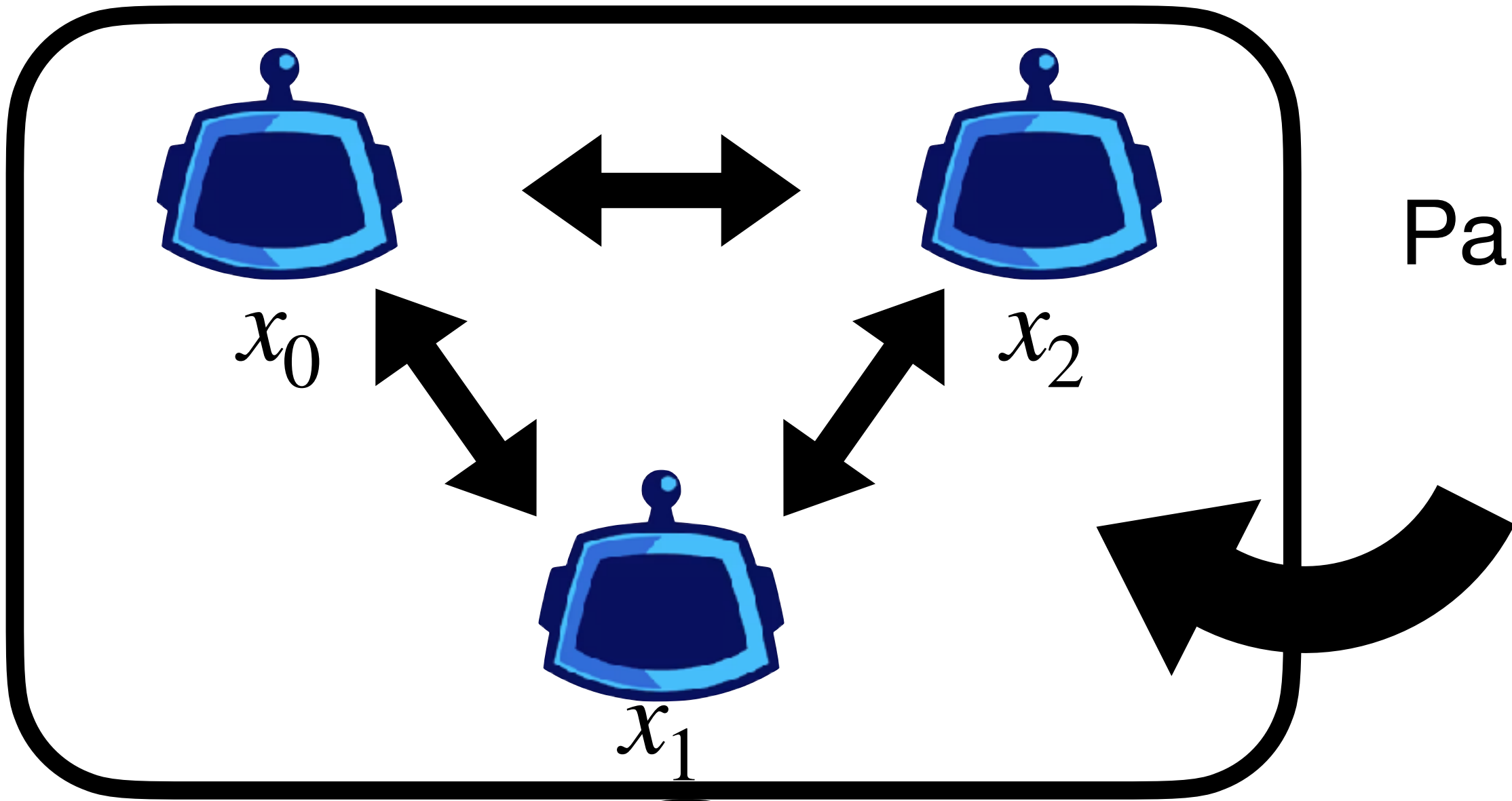
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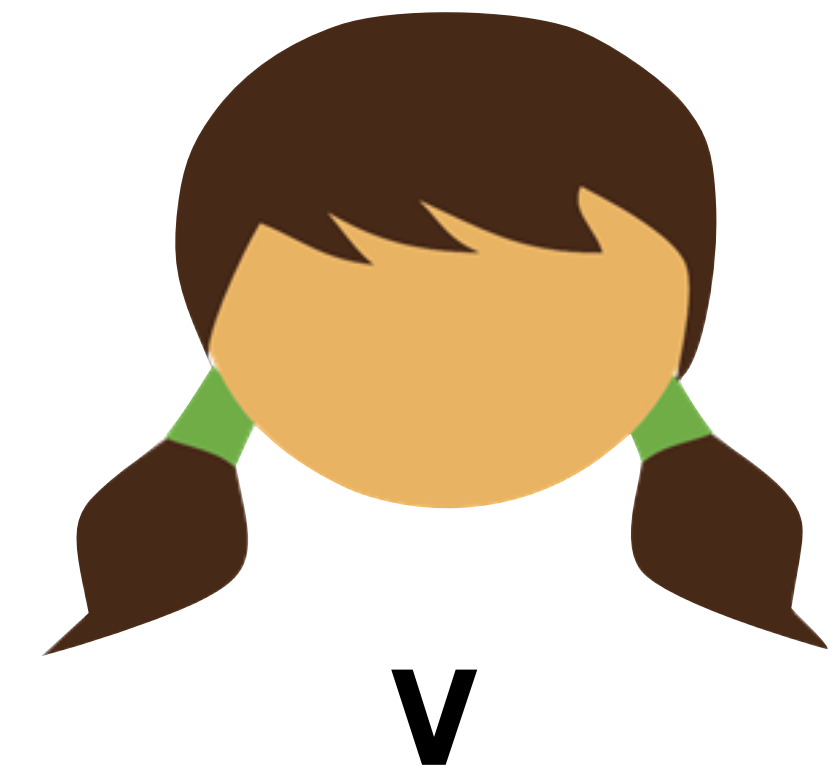
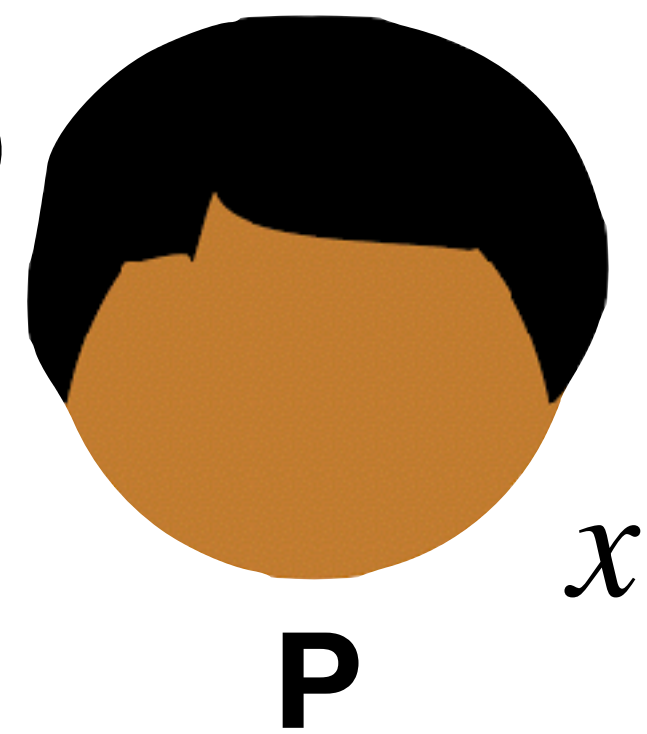
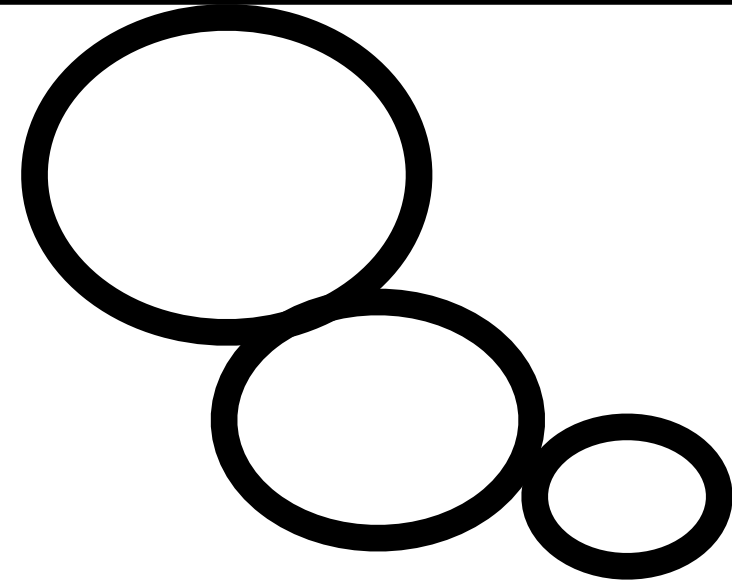


ZK from MPC in the Head

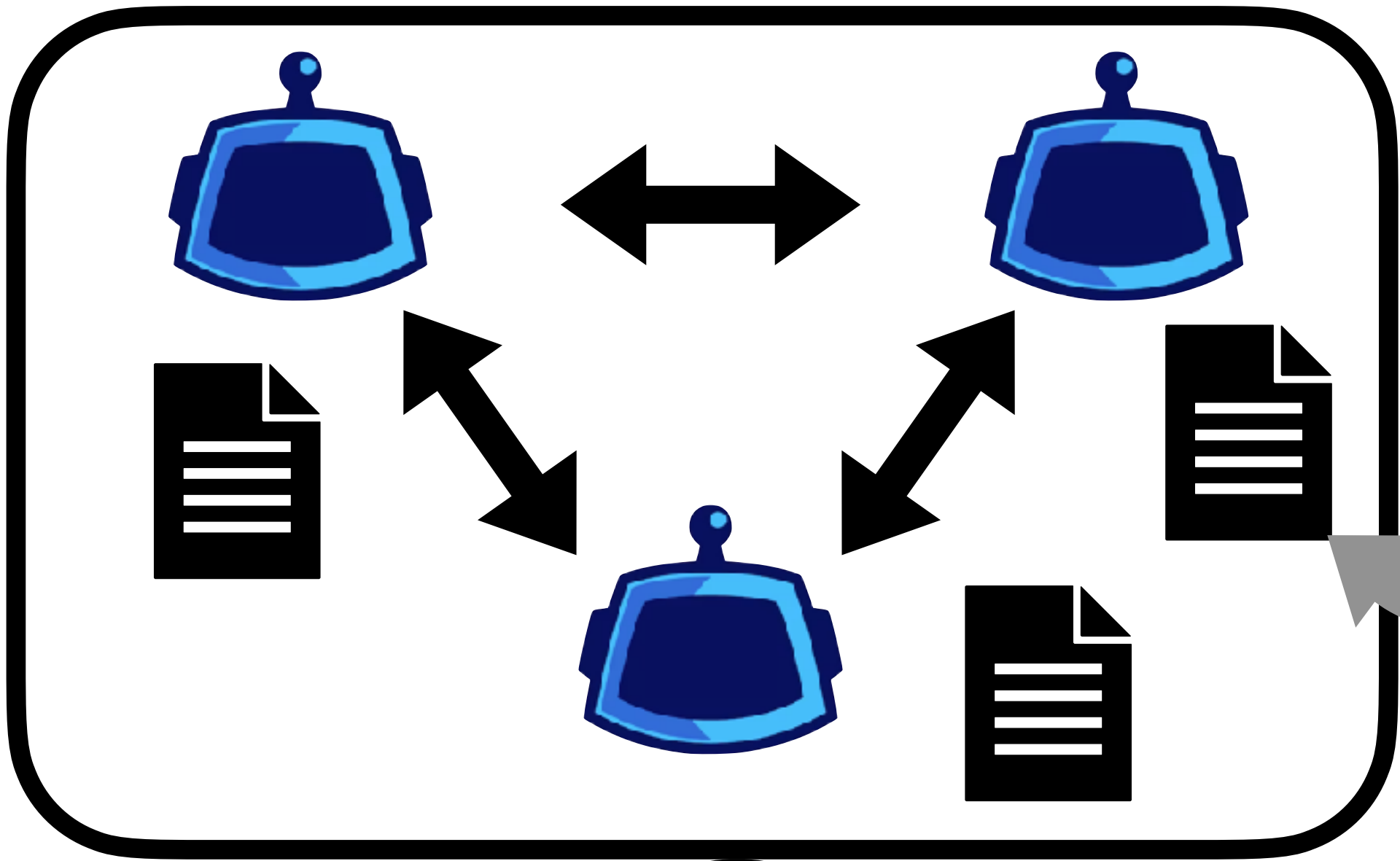


Parties compute $C(x_0 \oplus x_1 \oplus x_2)$
using the GMW protocol

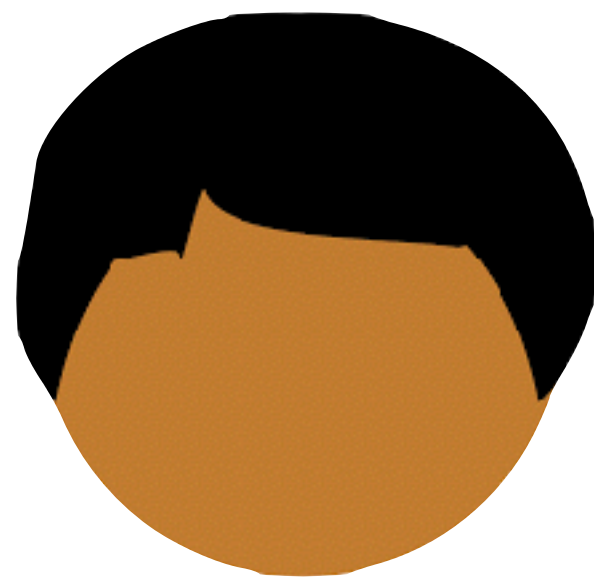
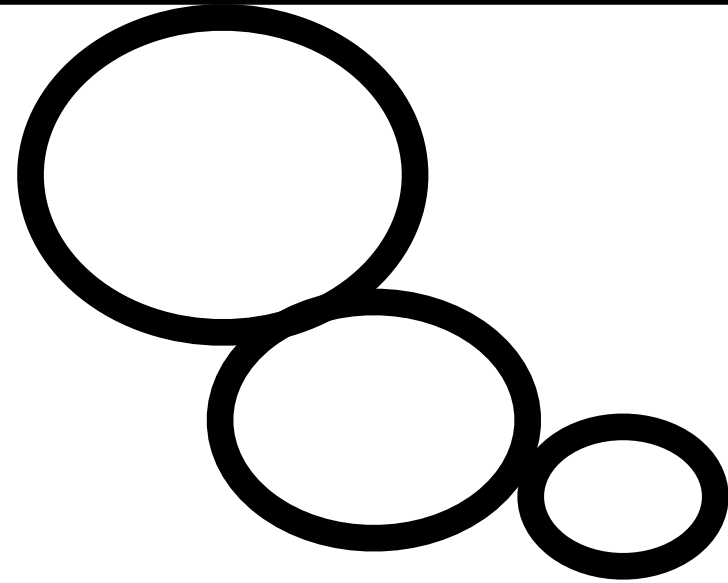
C



ZK from MPC in the Head



Protocol transcript from the perspective of virtual party P_i

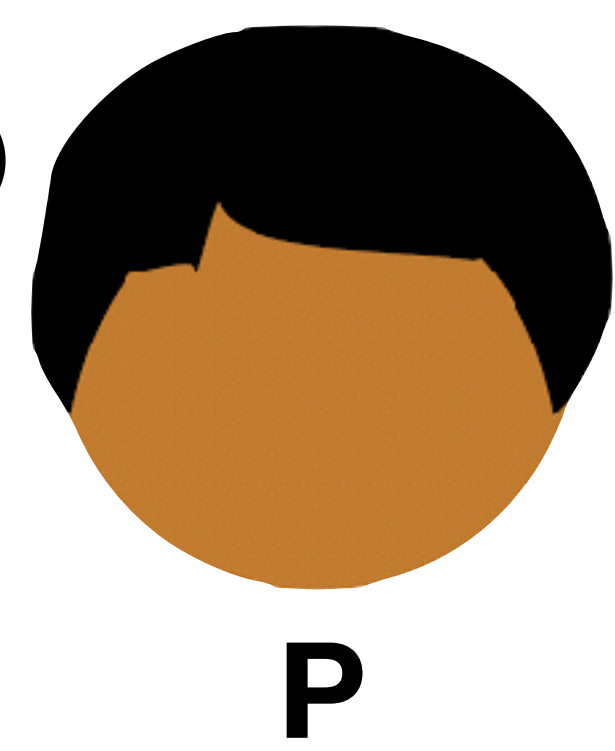
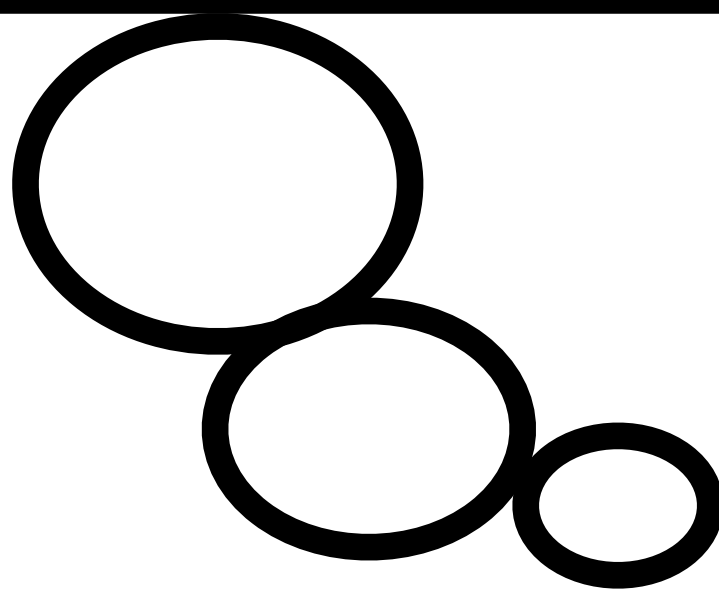
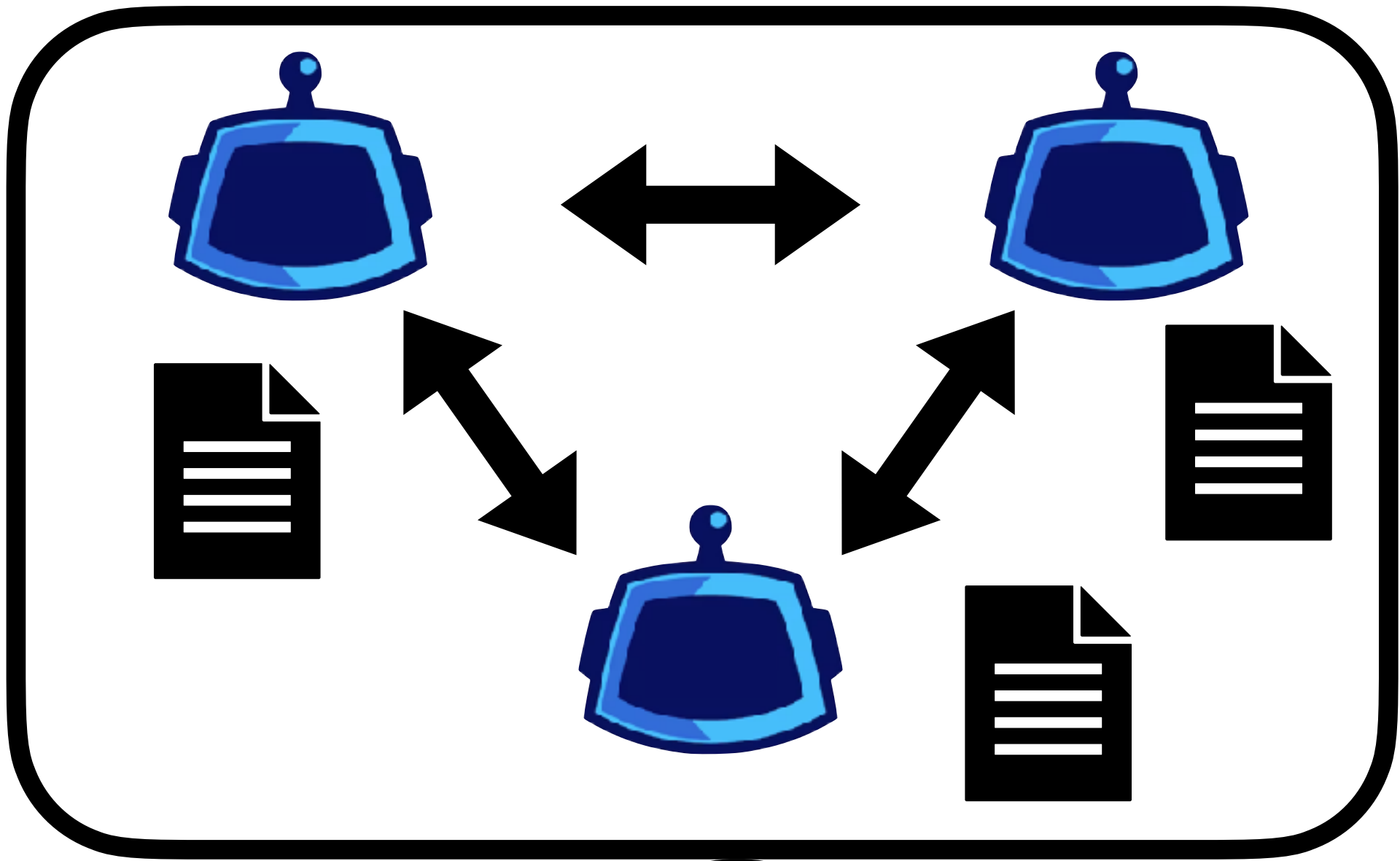


P



V

ZK from MPC in the Head



P

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

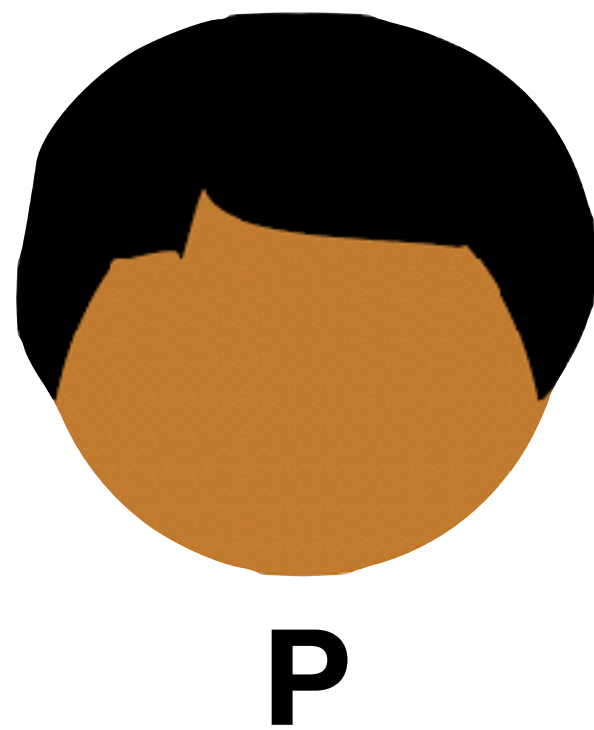
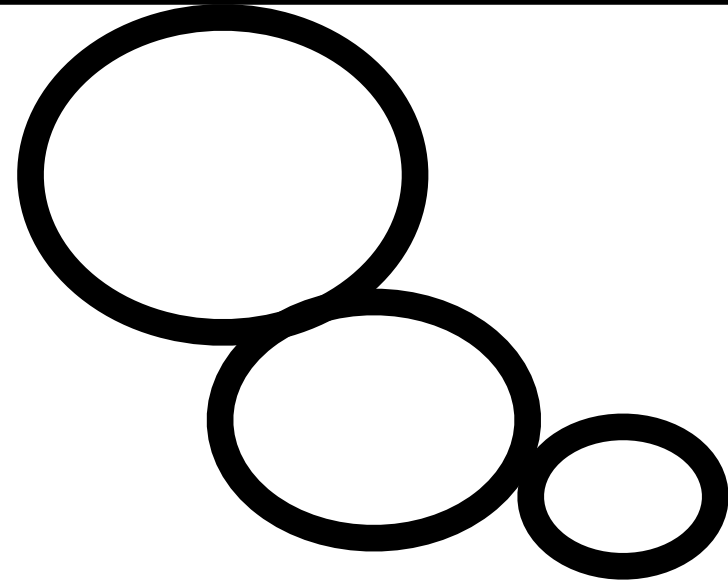
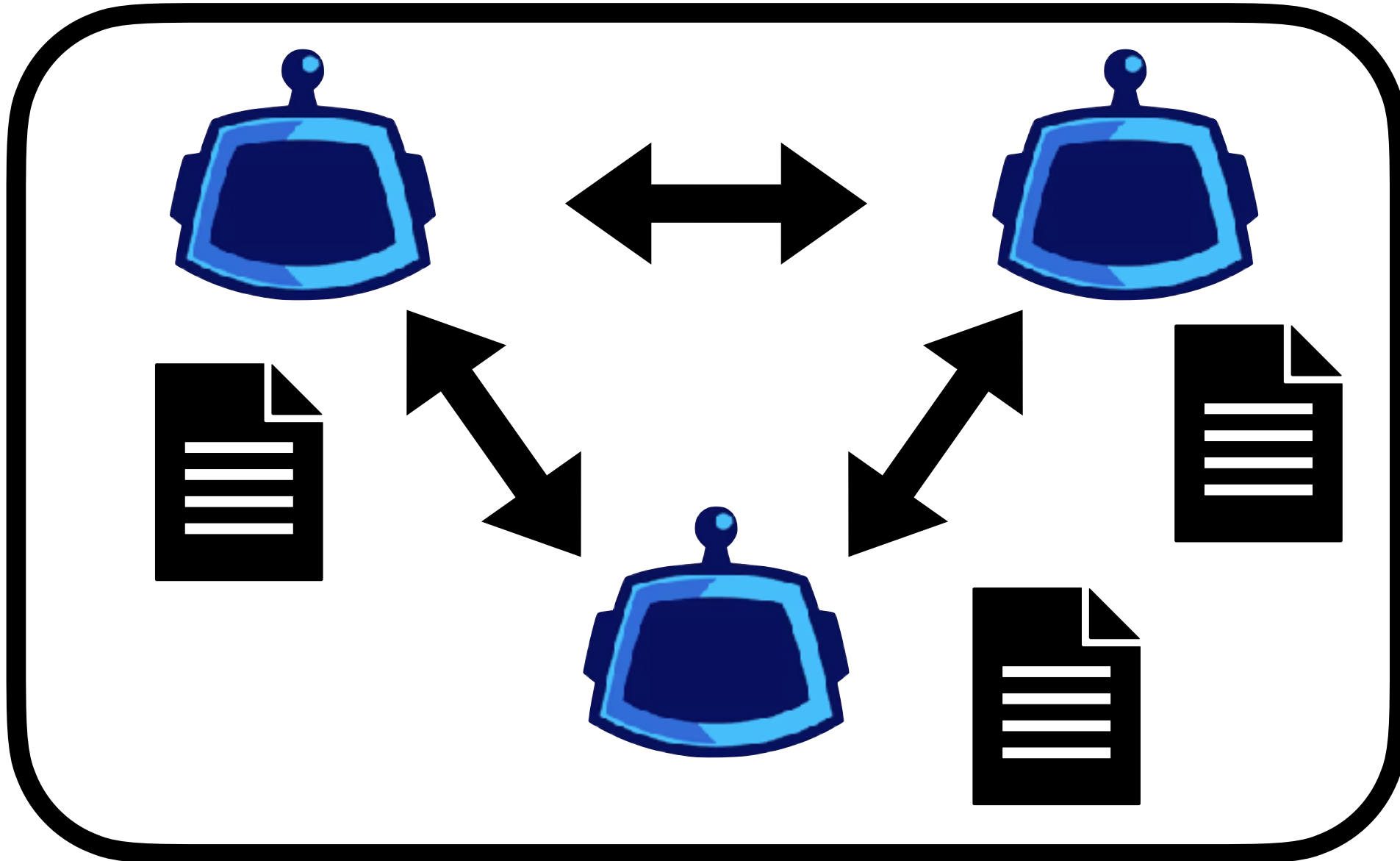


t_0 r_0 t_2 r_2

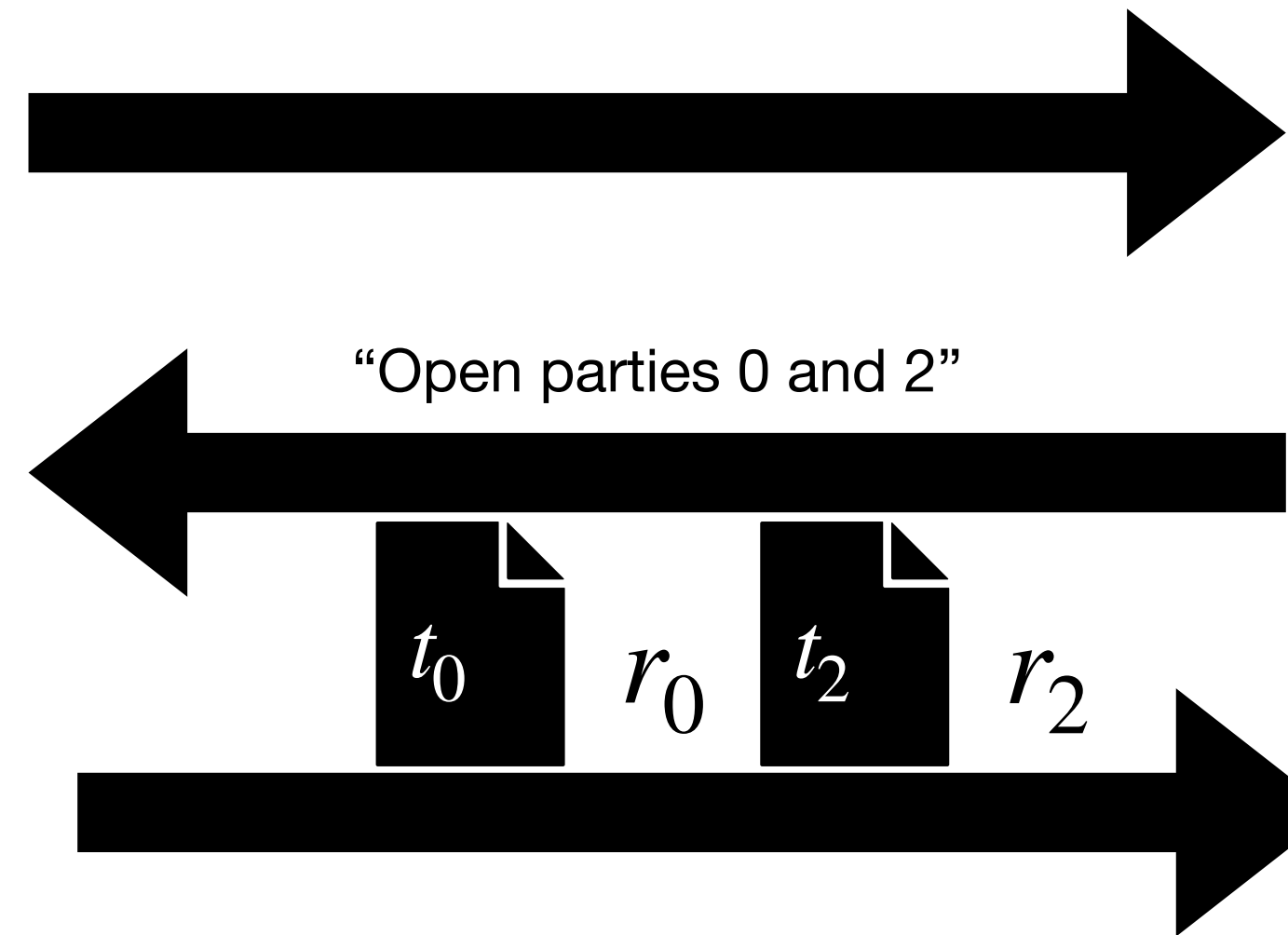


V

ZK from MPC in the Head

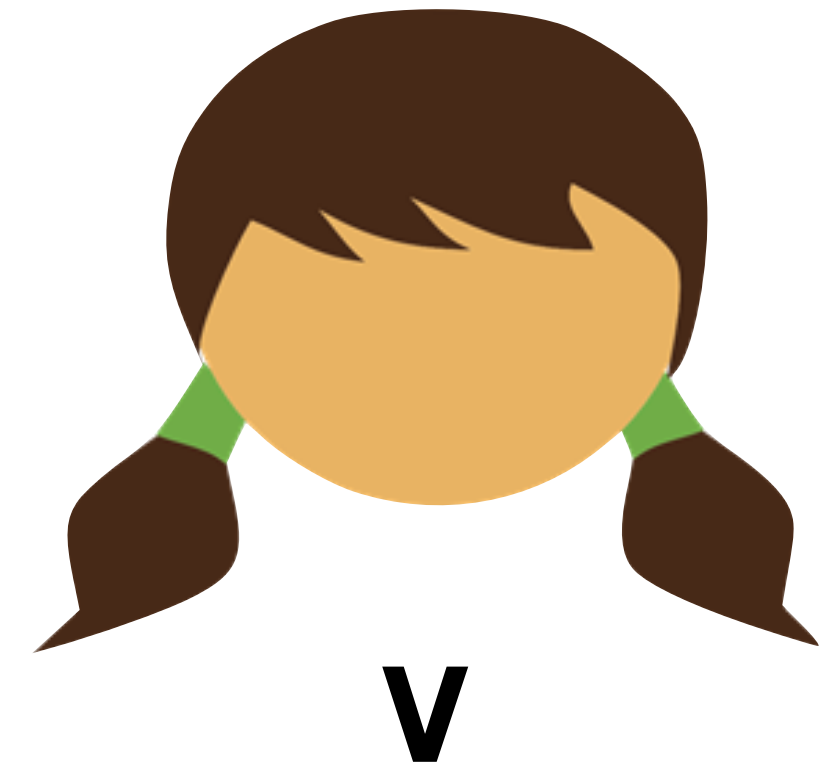


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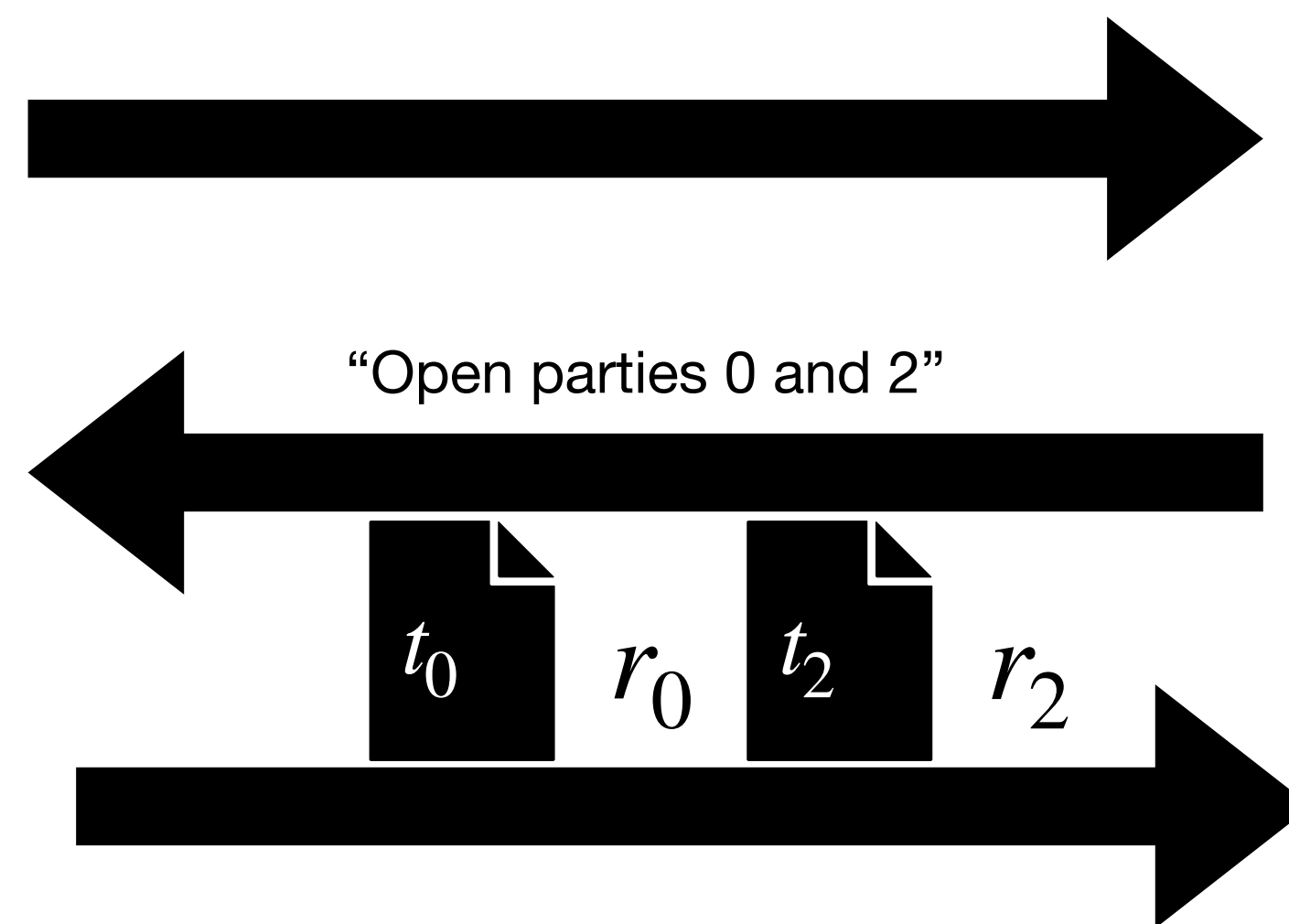
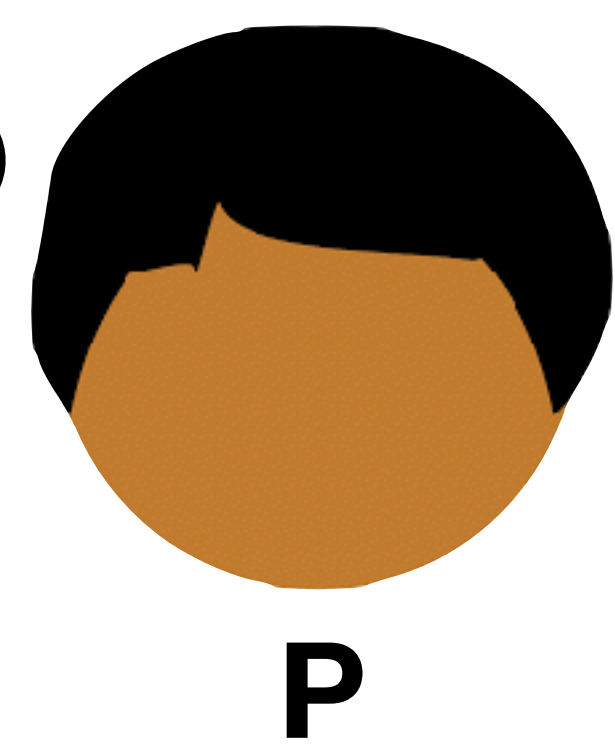
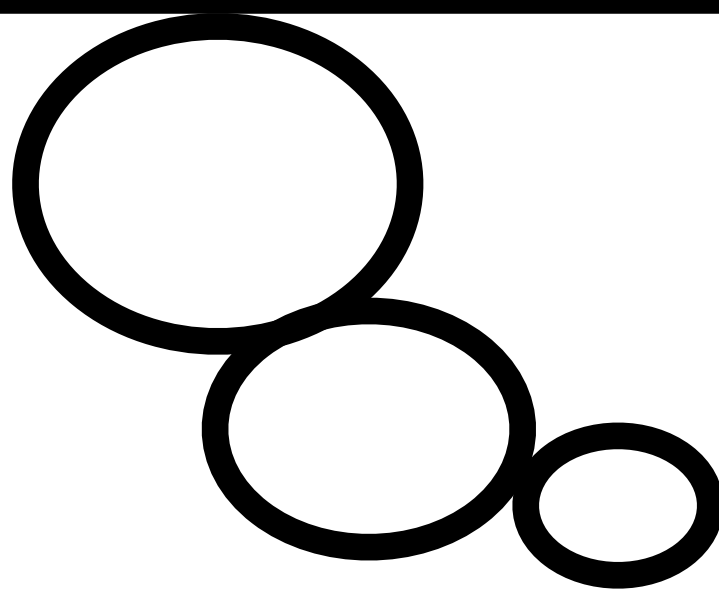
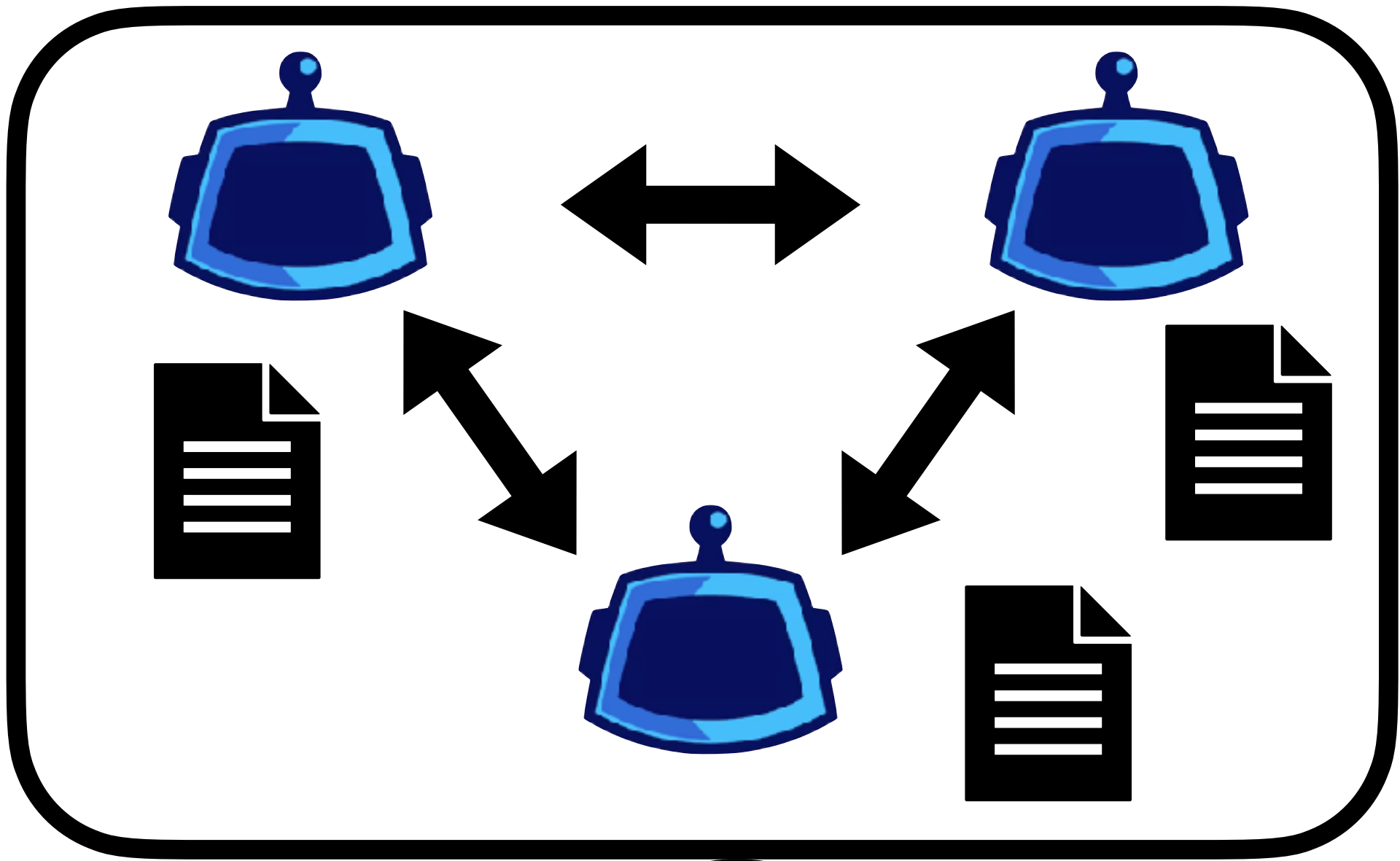
$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
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Completeness?



V

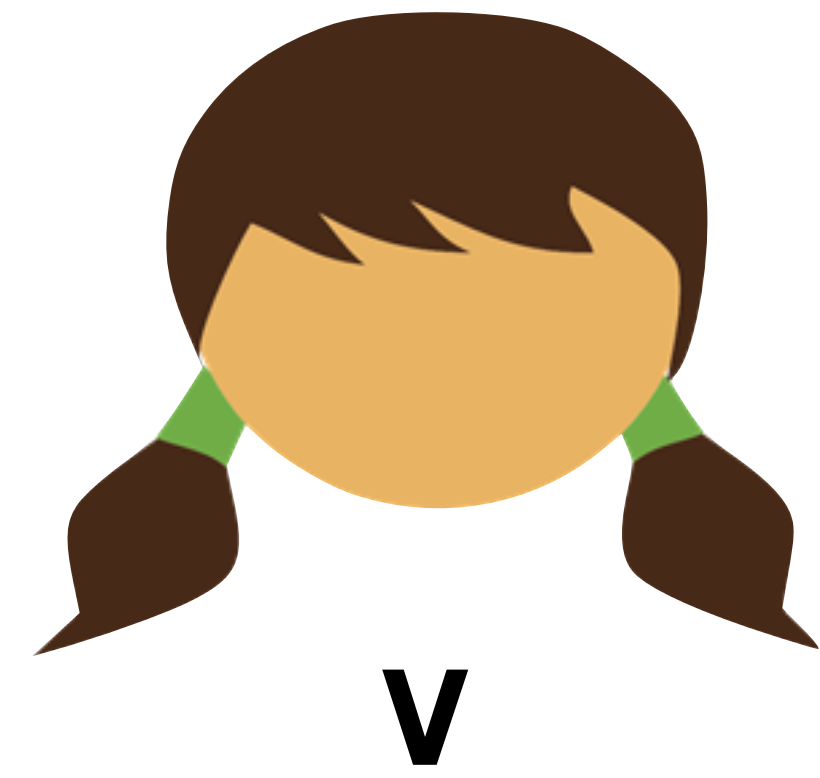
ZK from MPC in the Head



Completeness?

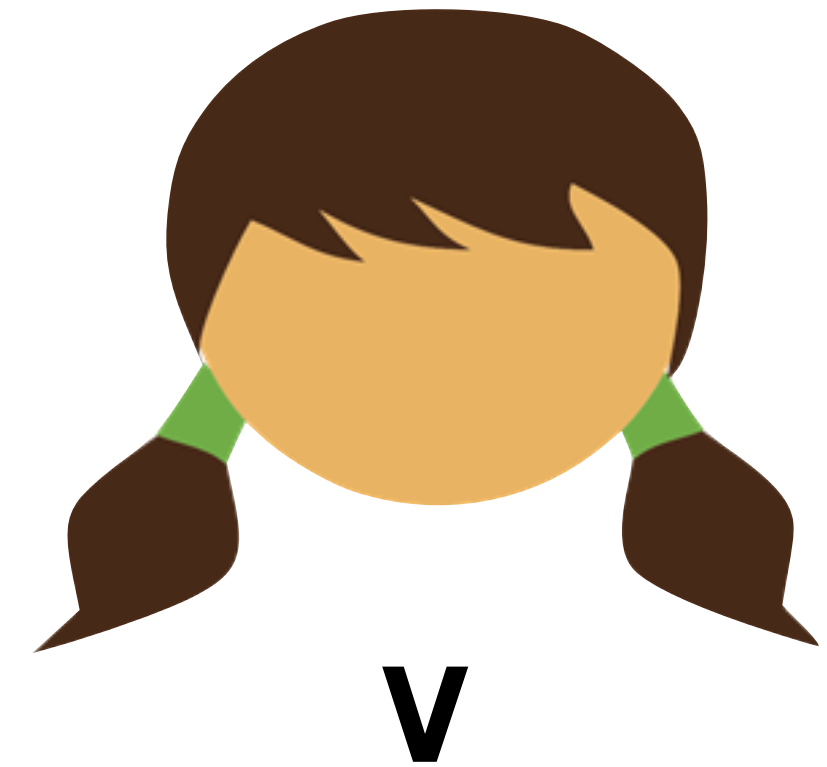
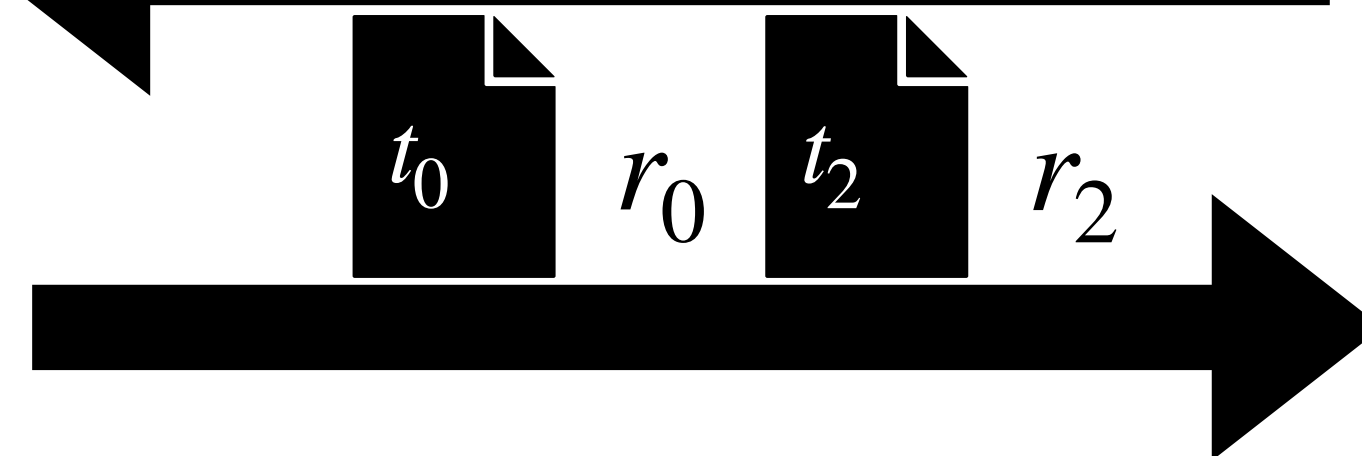
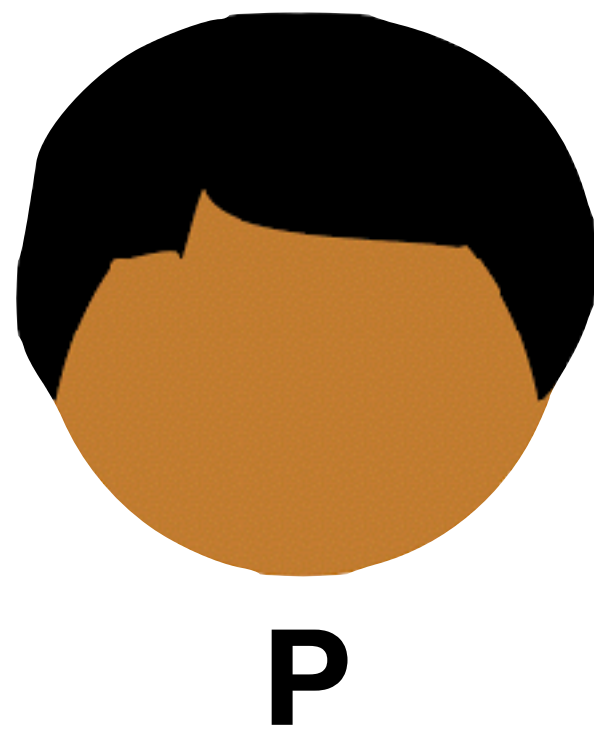
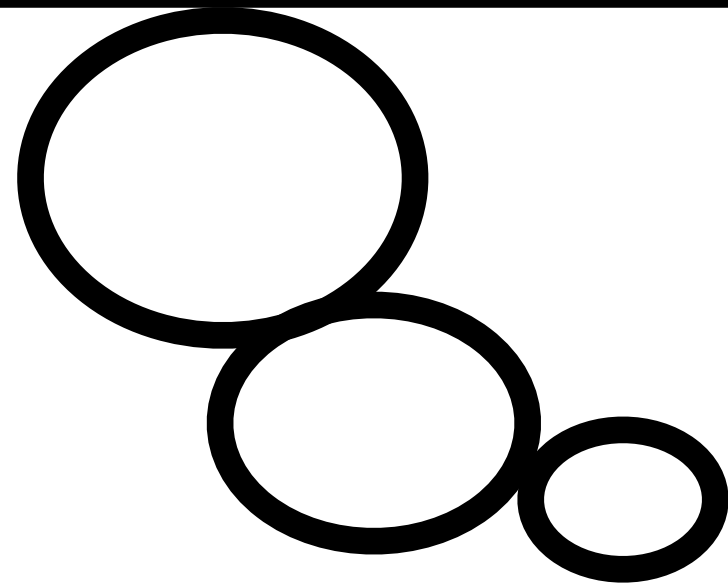
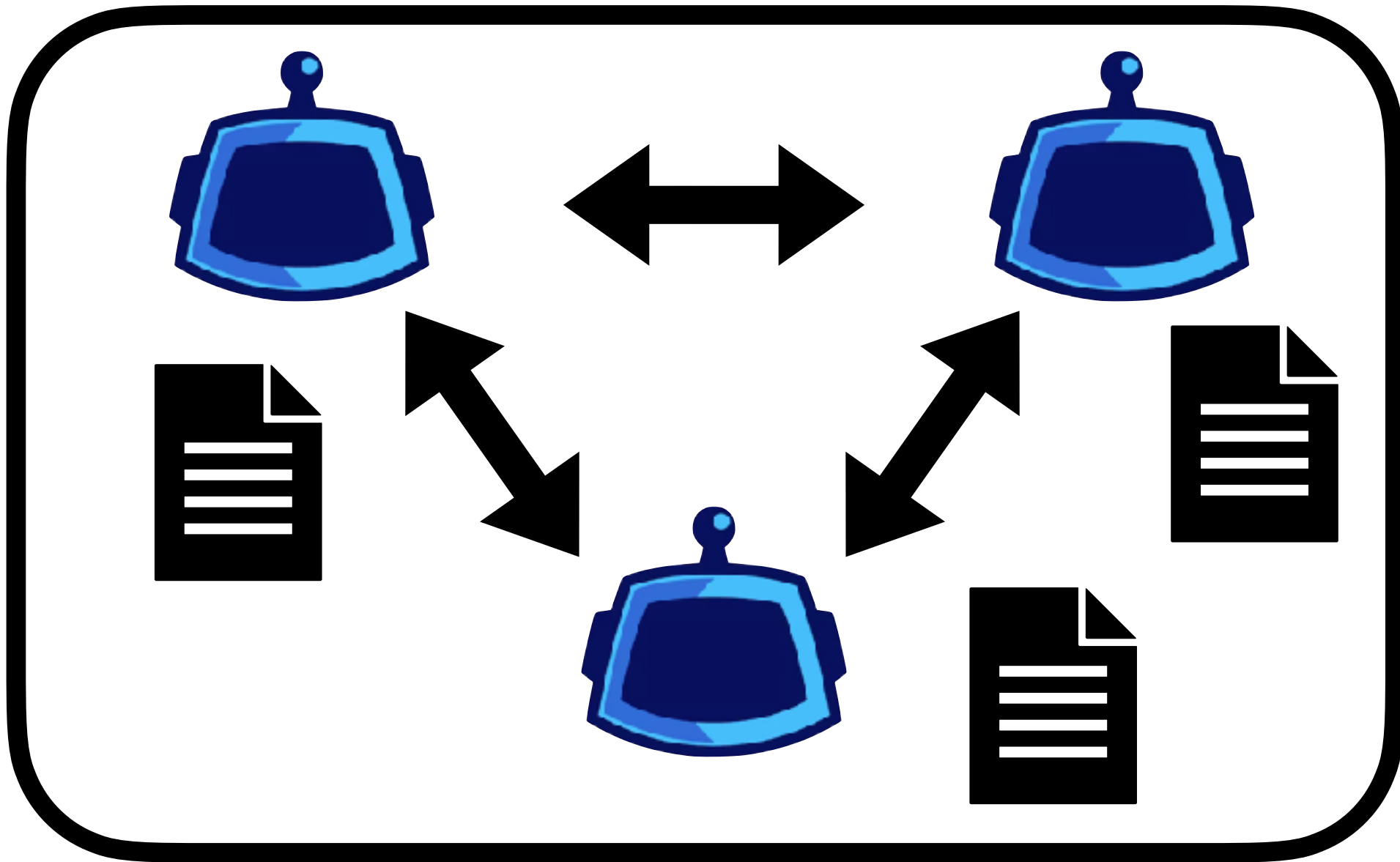
Correctness of GMW

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$



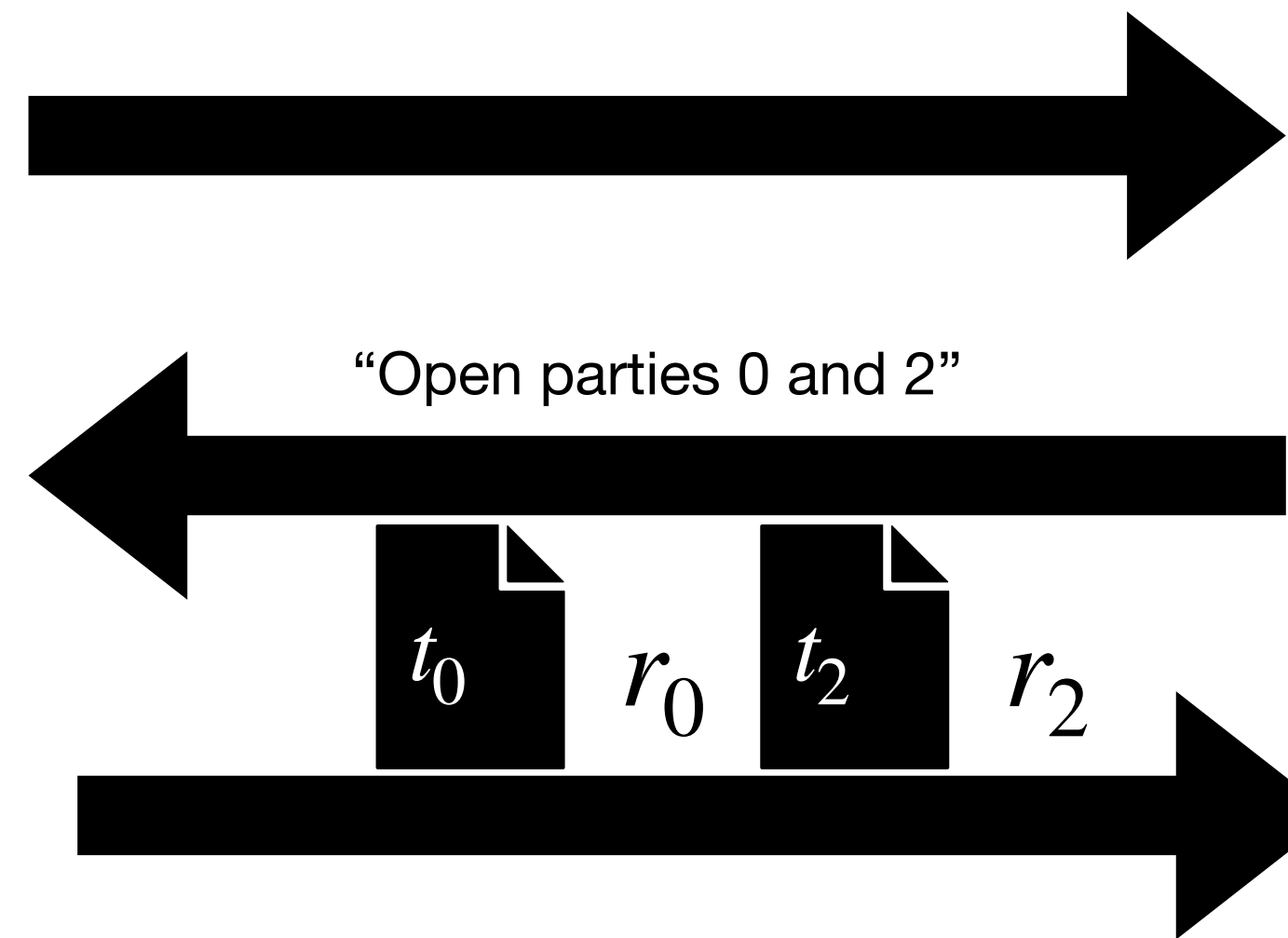
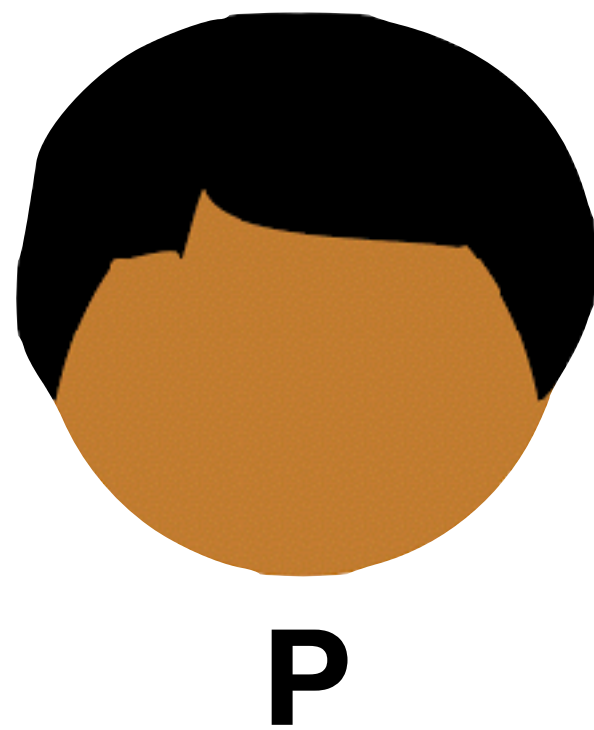
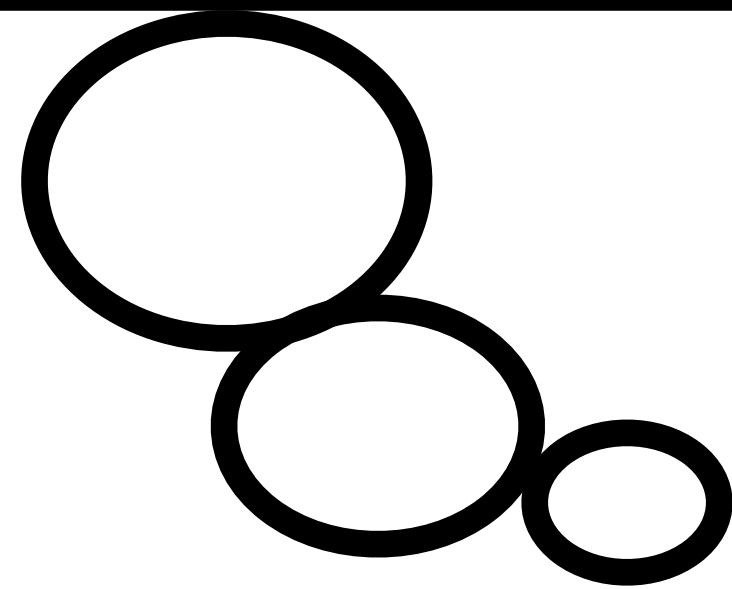
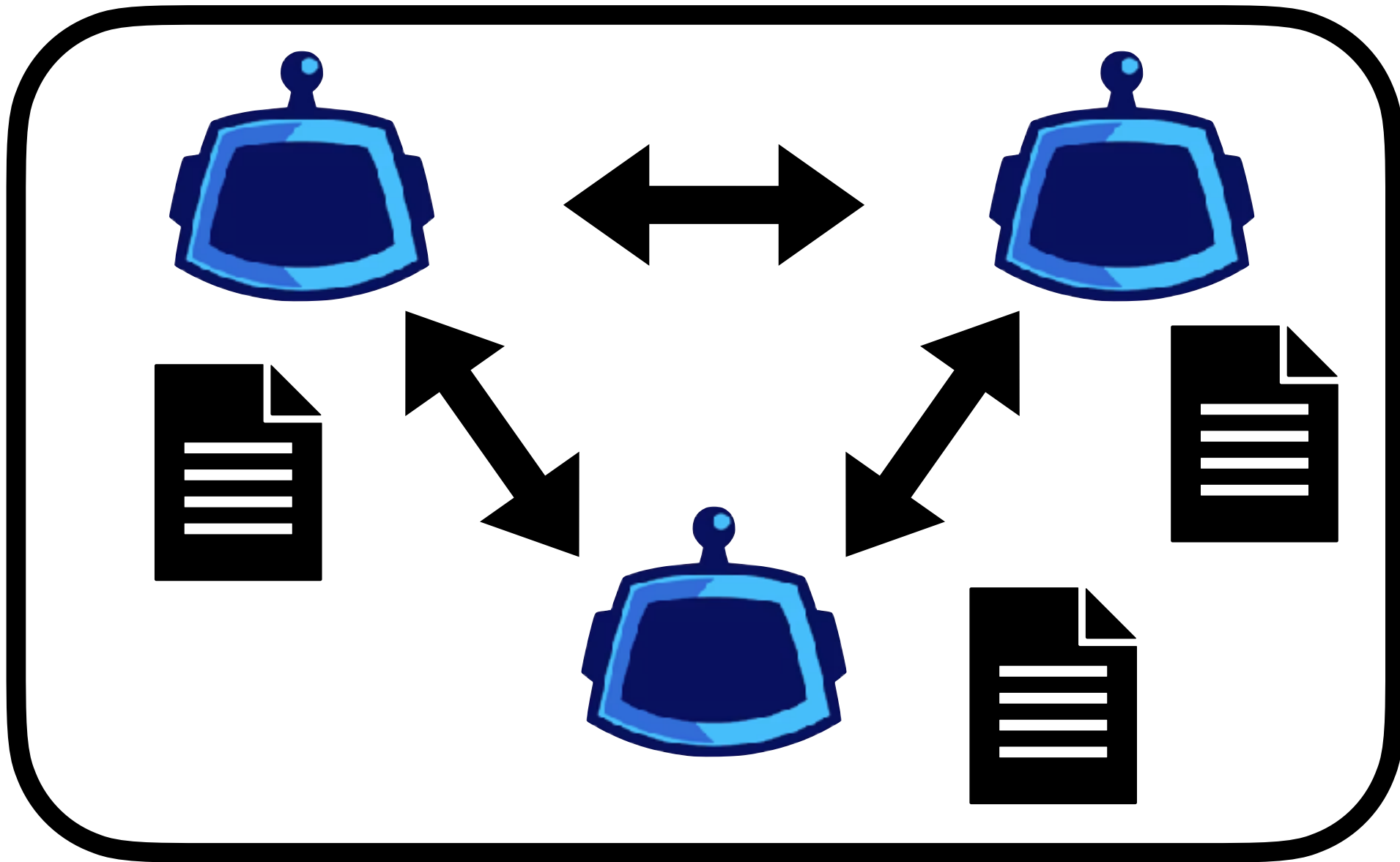
ZK from MPC in the Head

Zero Knowledge?



$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

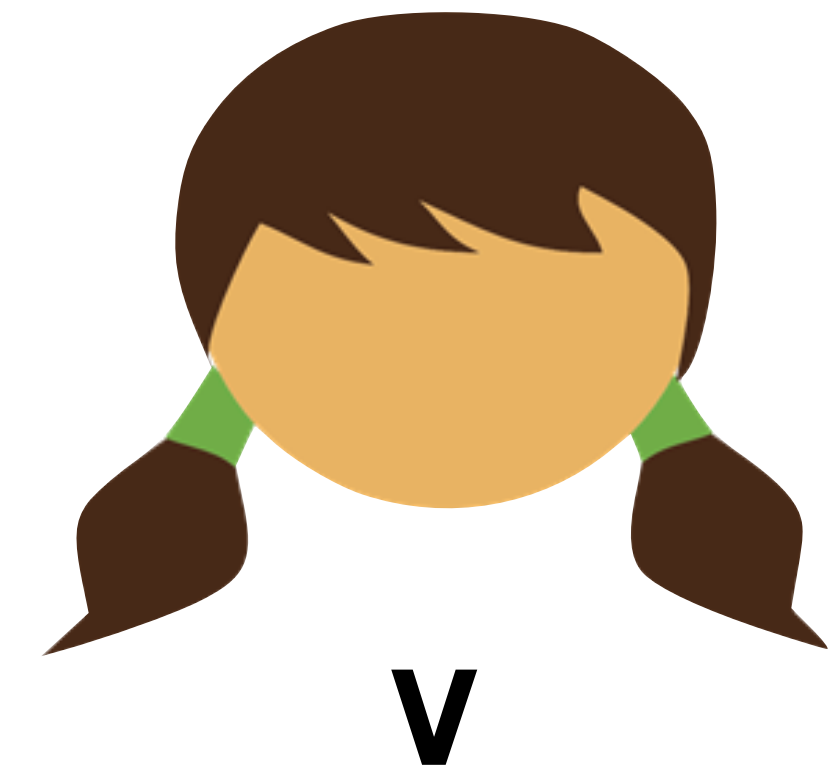
ZK from MPC in the Head



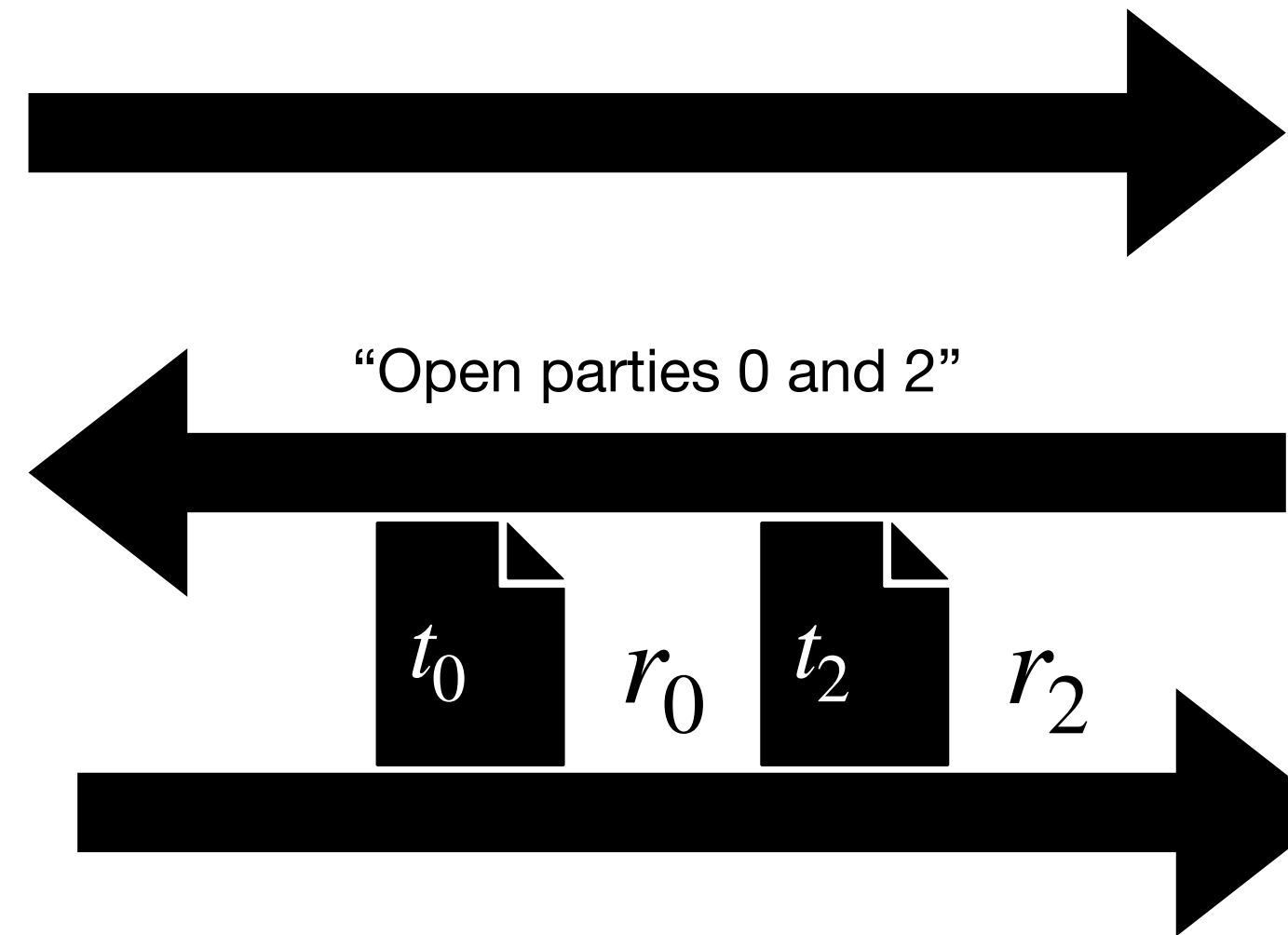
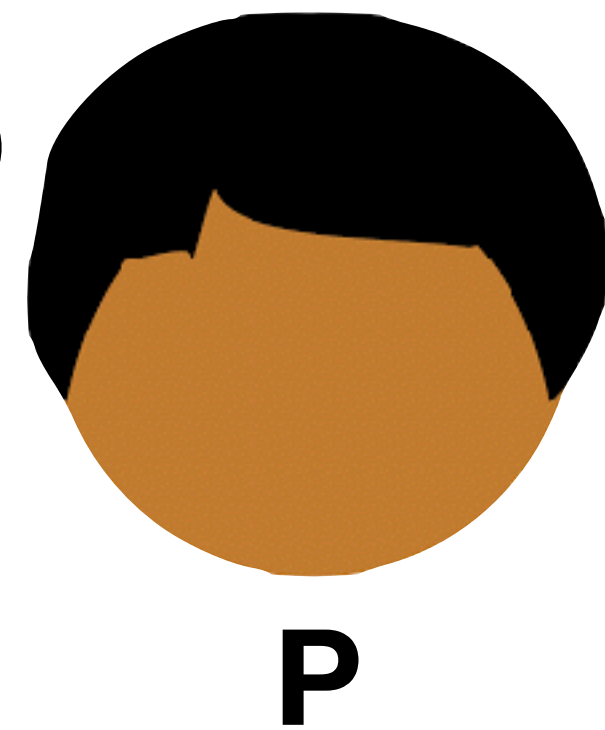
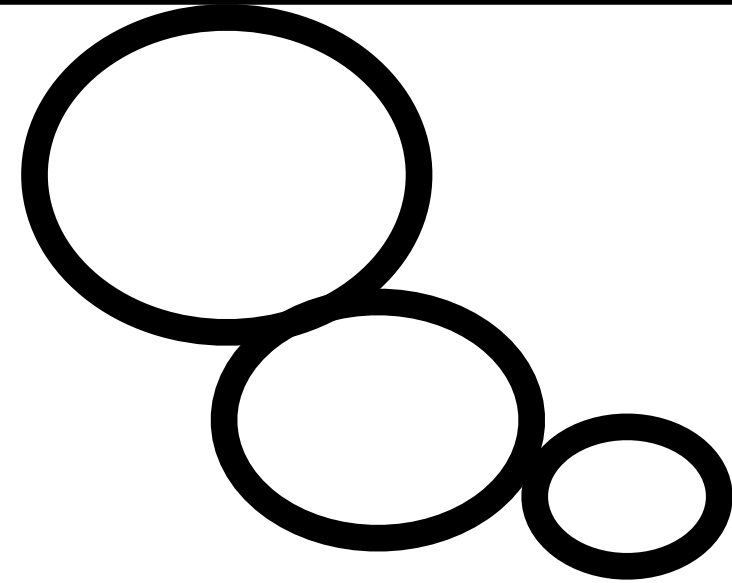
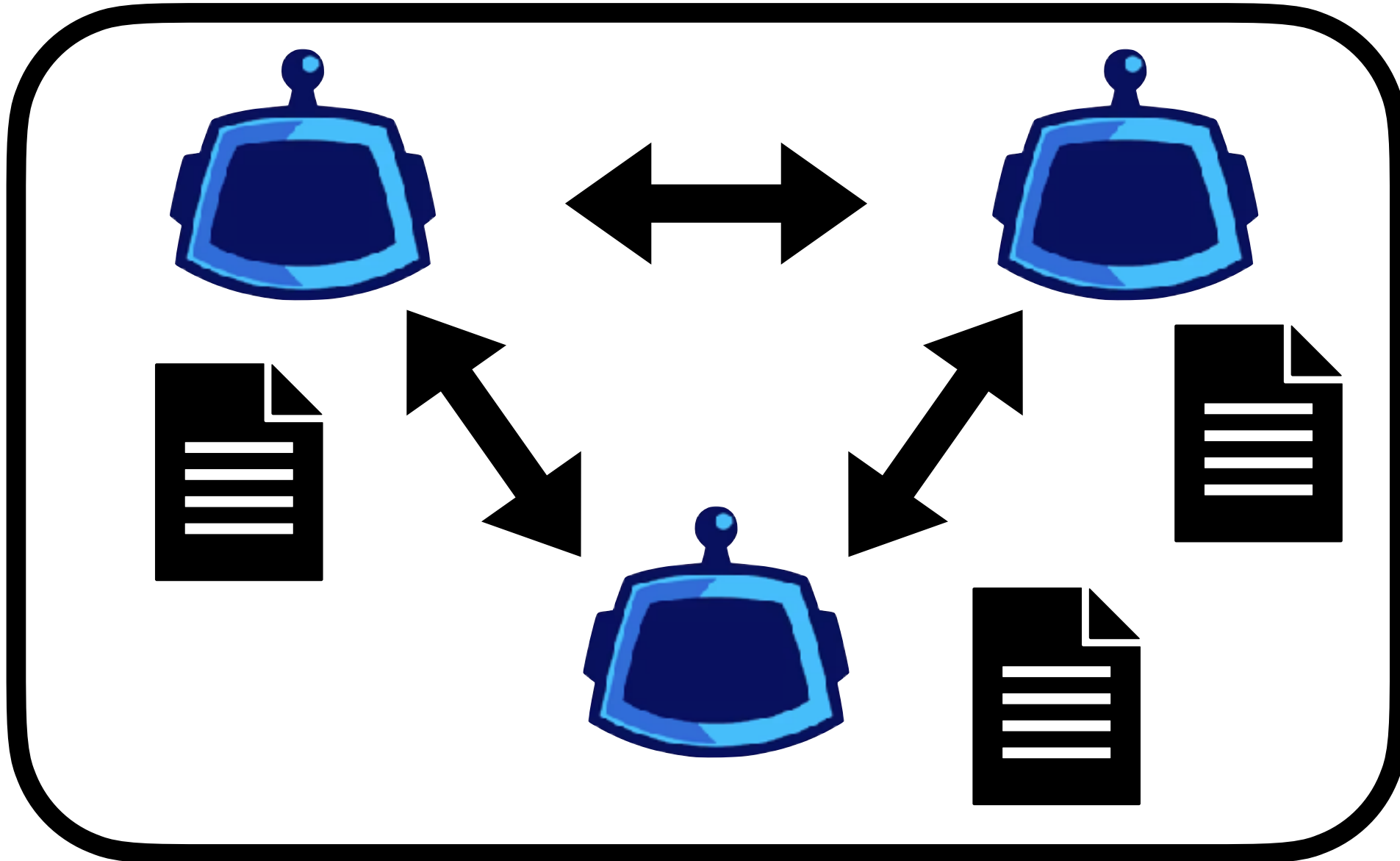
Zero Knowledge?

Security of GMW

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

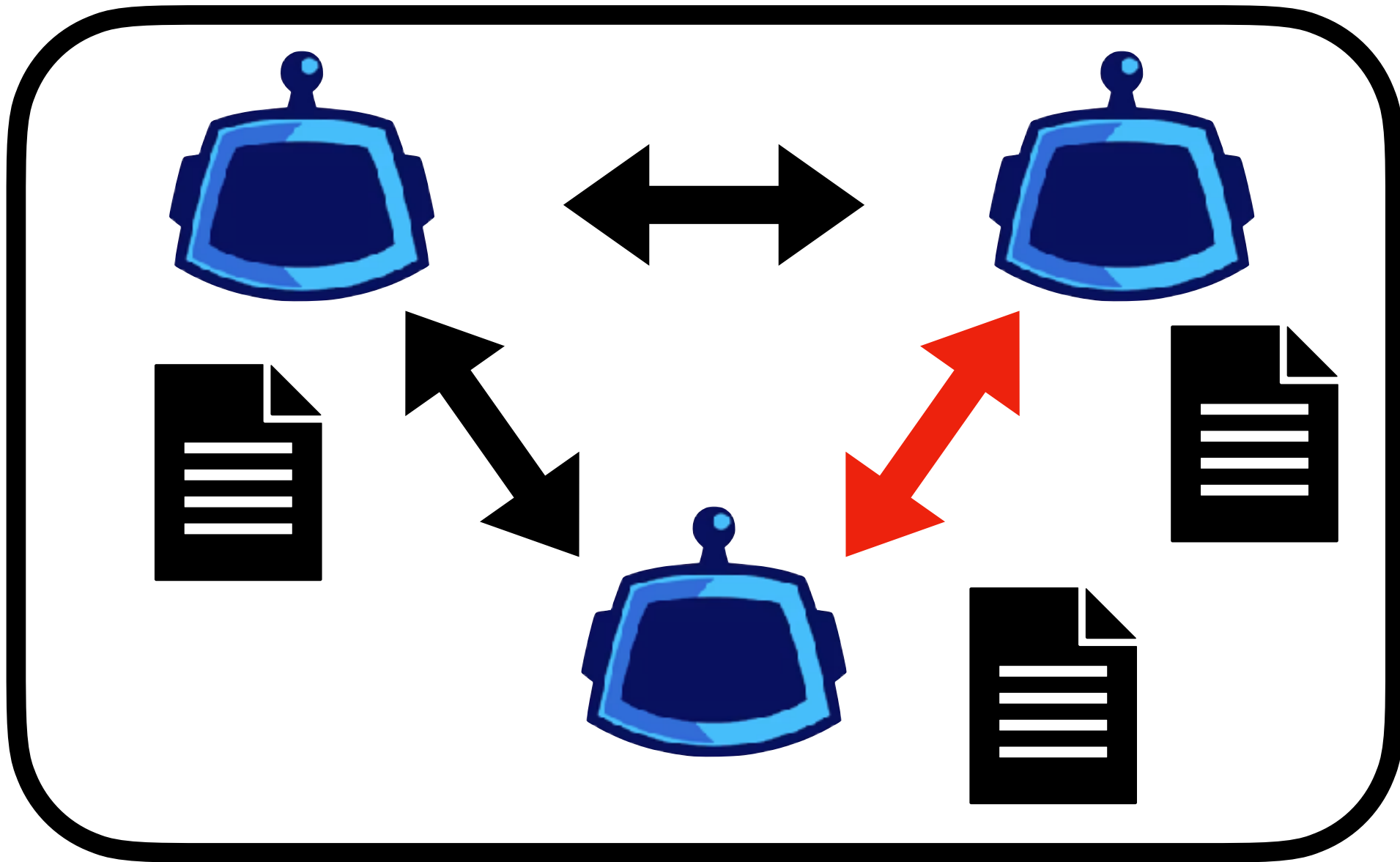


ZK from MPC in the Head



Soundness?

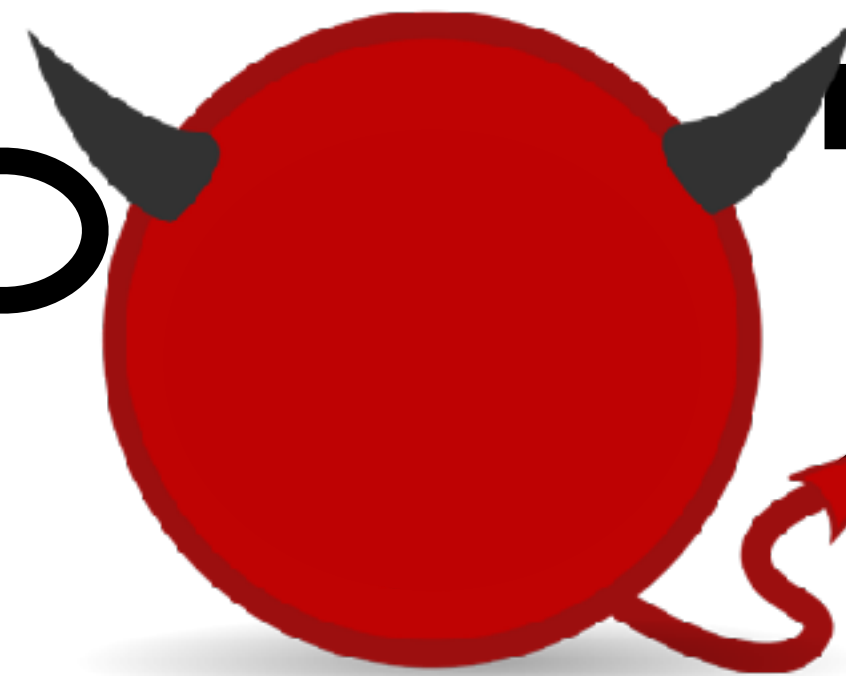
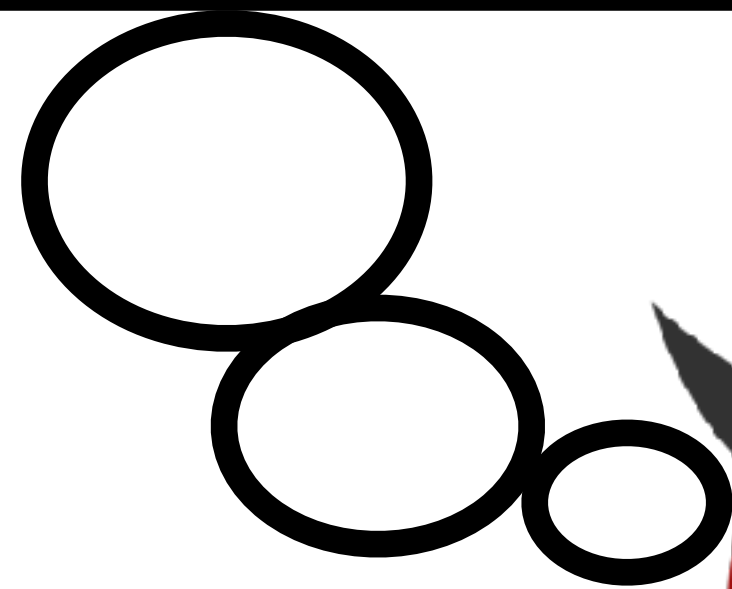
ZK from MPC in the Head



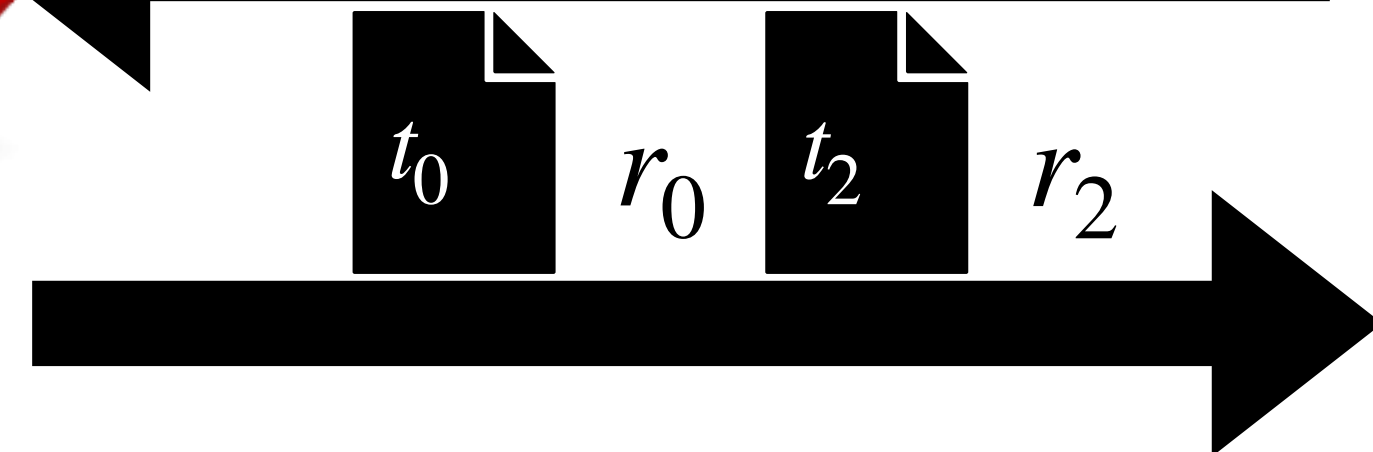
Soundness?

To cheat, P must corrupt at least one edge (i.e., one party receives a message that was not sent by the other)

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

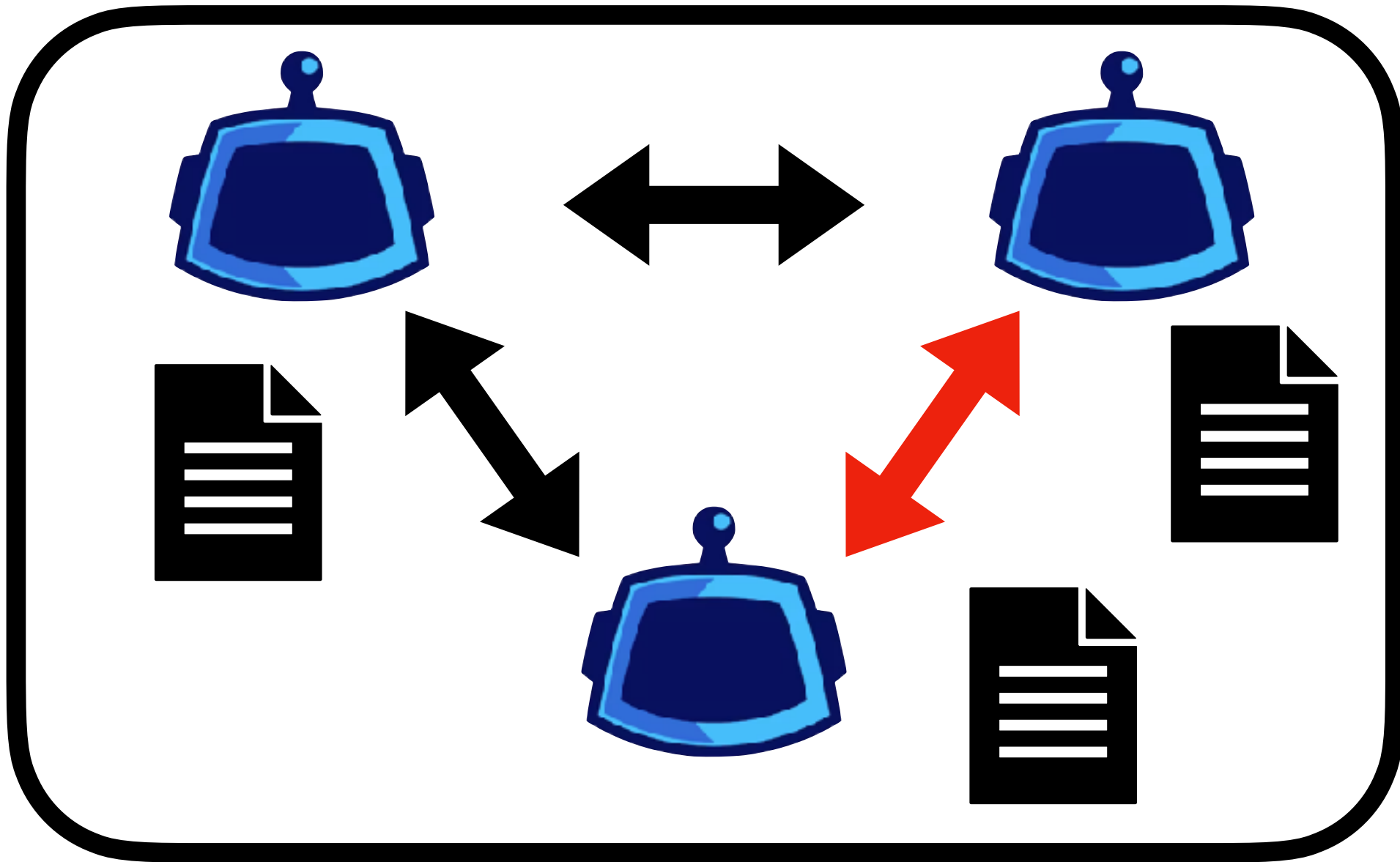


P



V

ZK from MPC in the Head

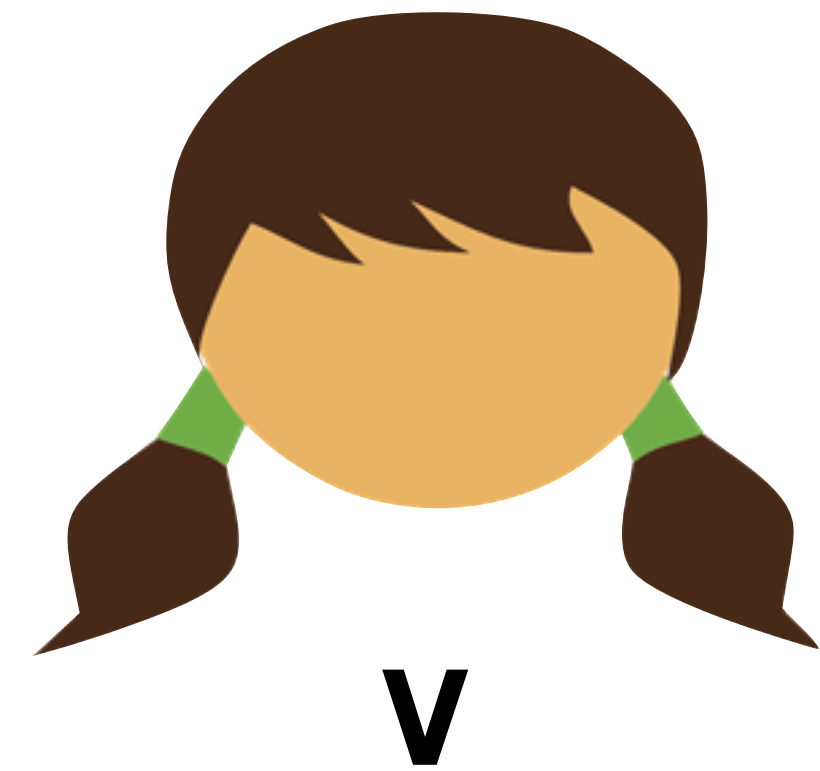
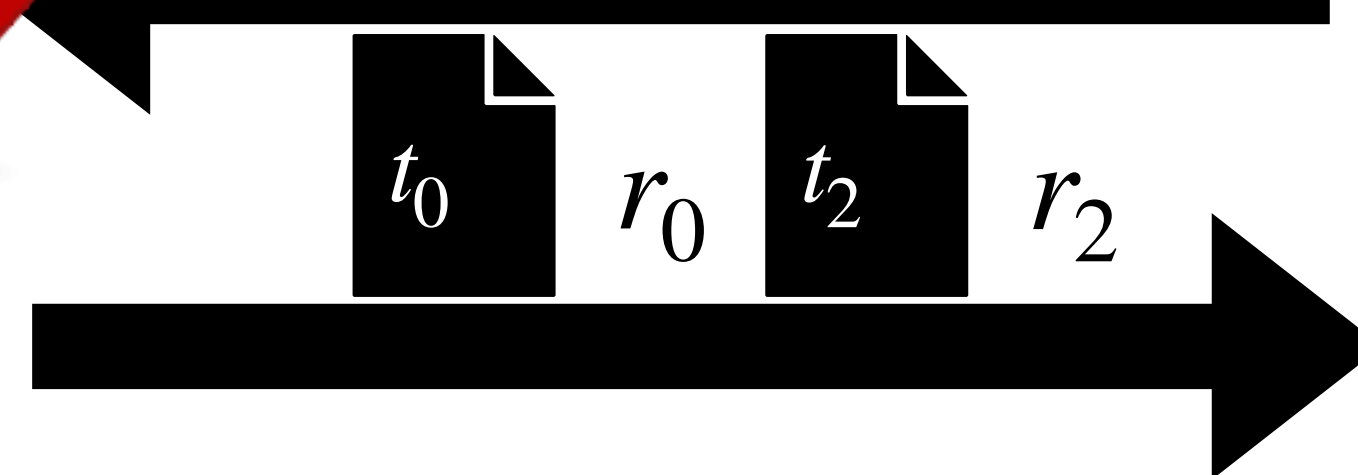
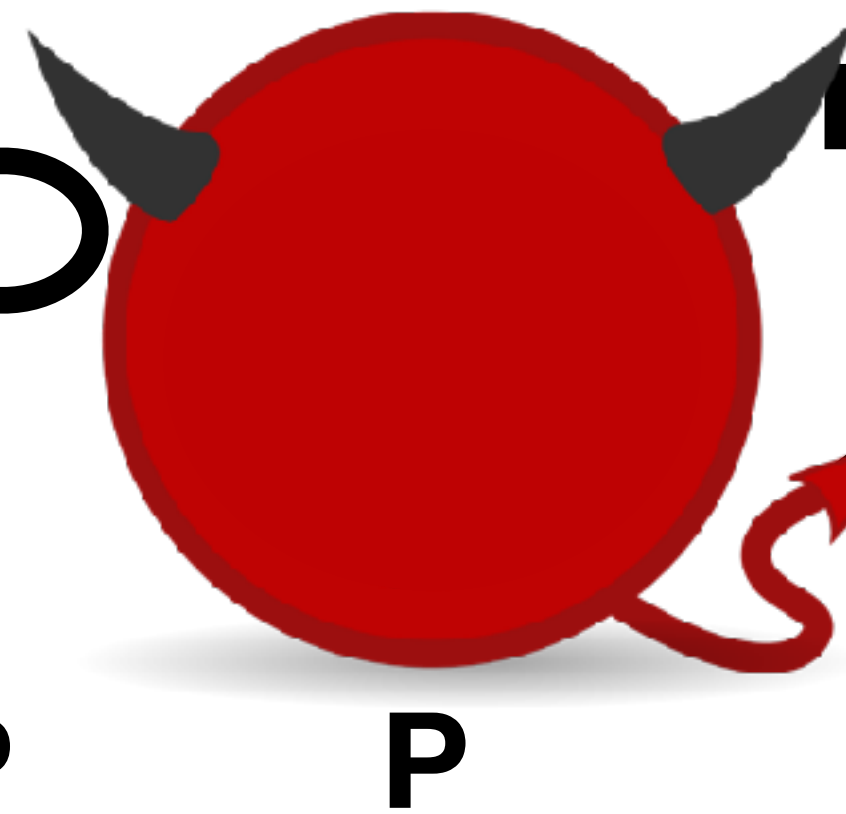
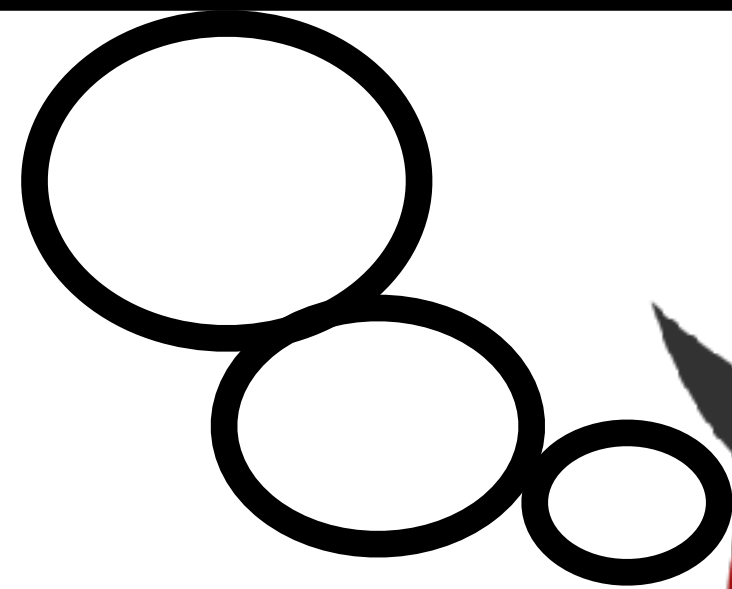


Soundness?

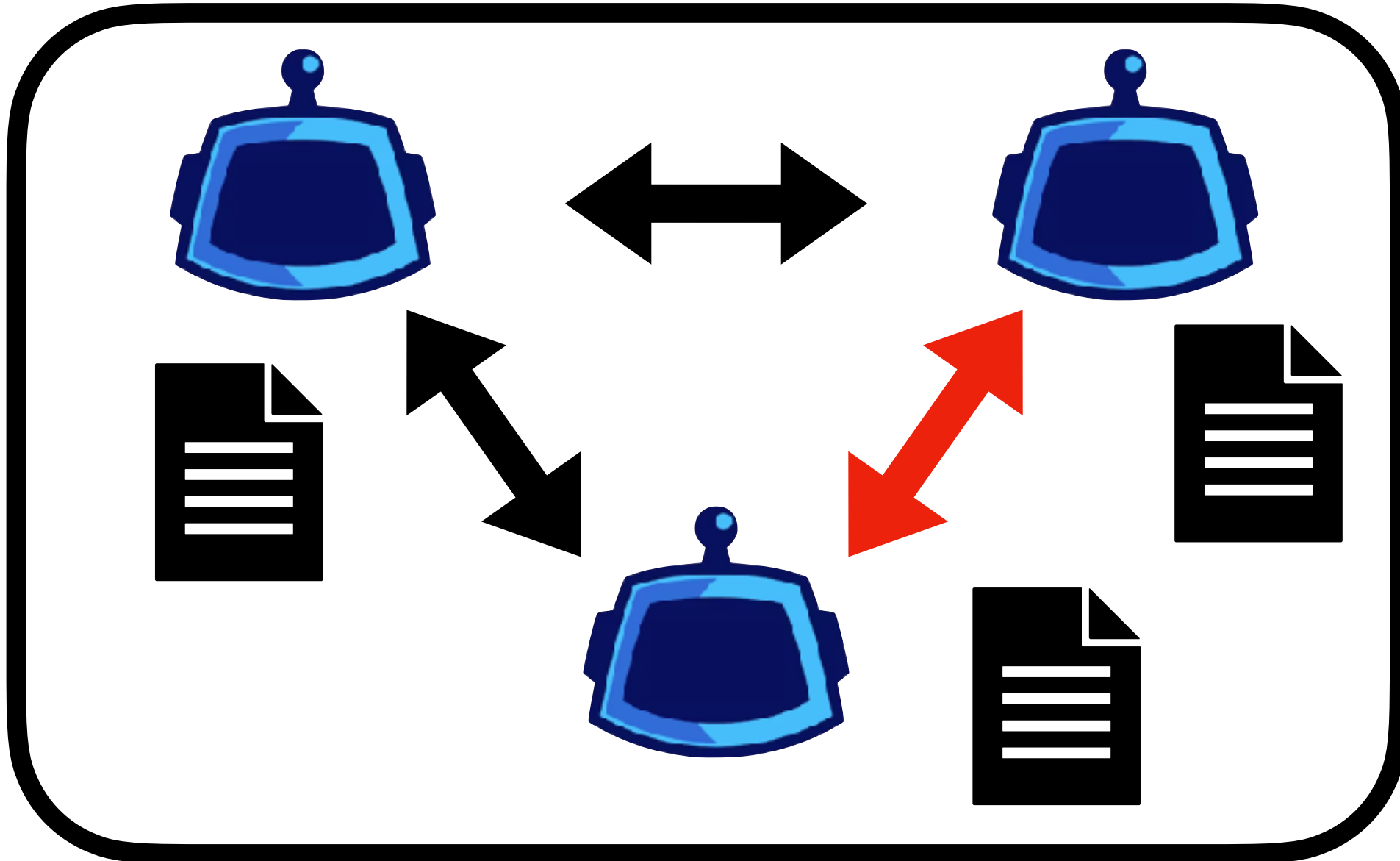
To cheat, P must corrupt at least one edge (i.e., one party receives a message that was not sent by the other)

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

By opening an edge, V has probability at least 1/3 to catch cheating P



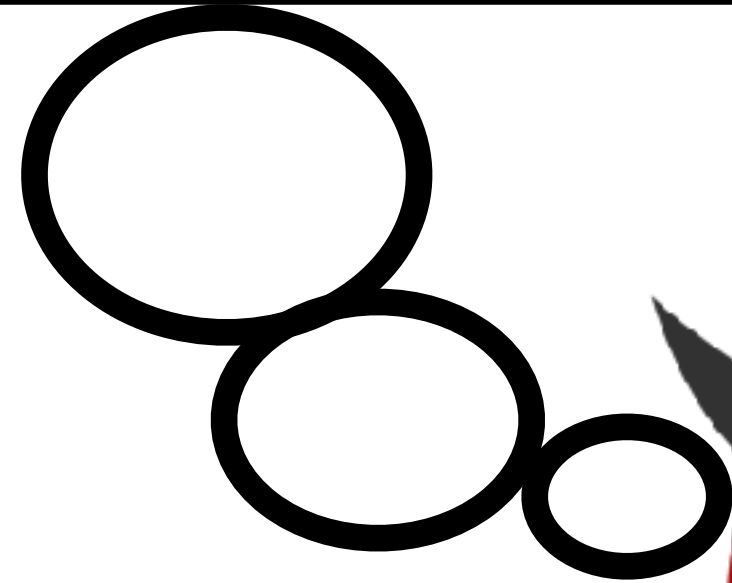
ZK from MPC in the Head



Soundness?

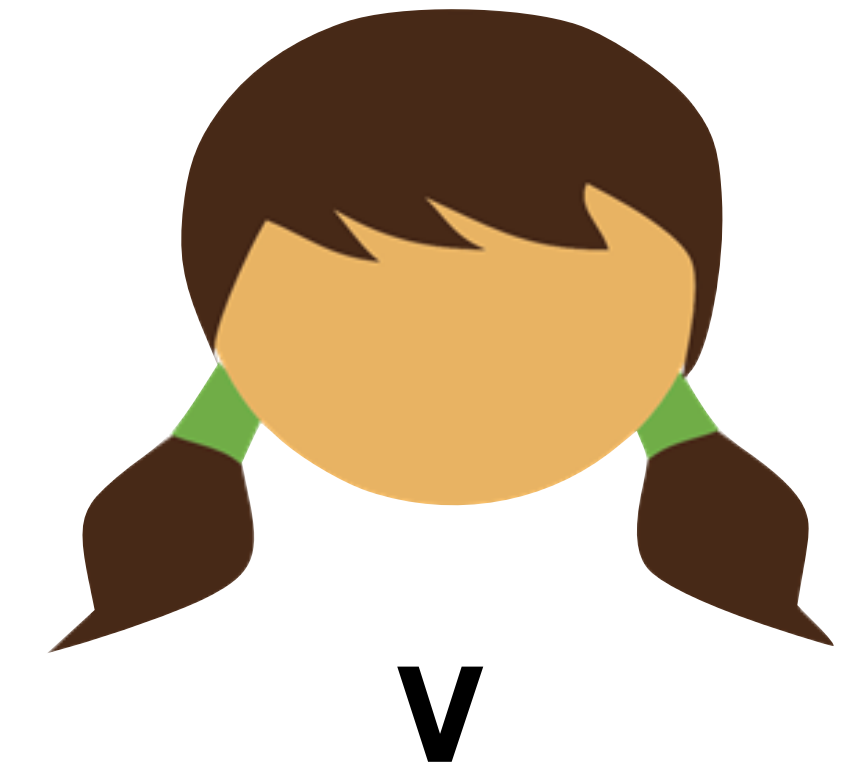
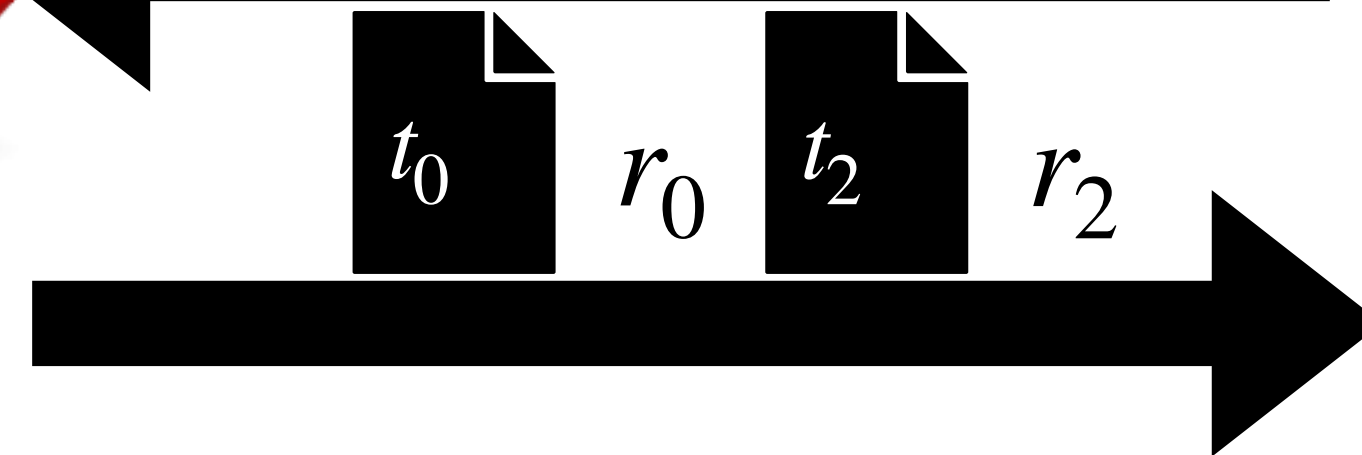
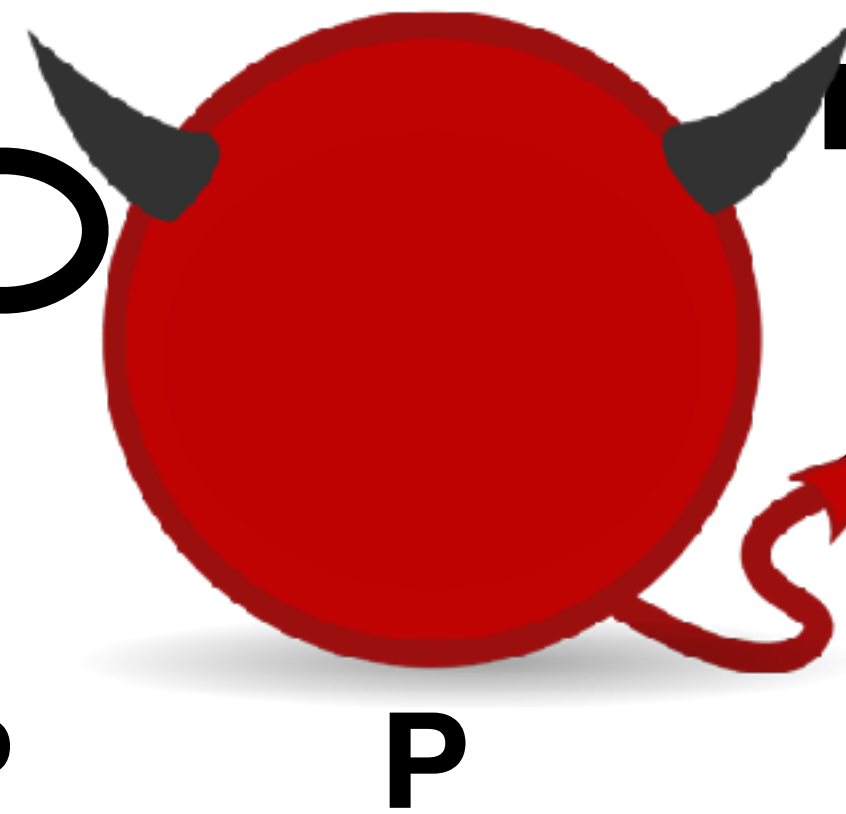
To cheat, P must corrupt at least one edge (i.e., one party receives a message that was not sent by the other)

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

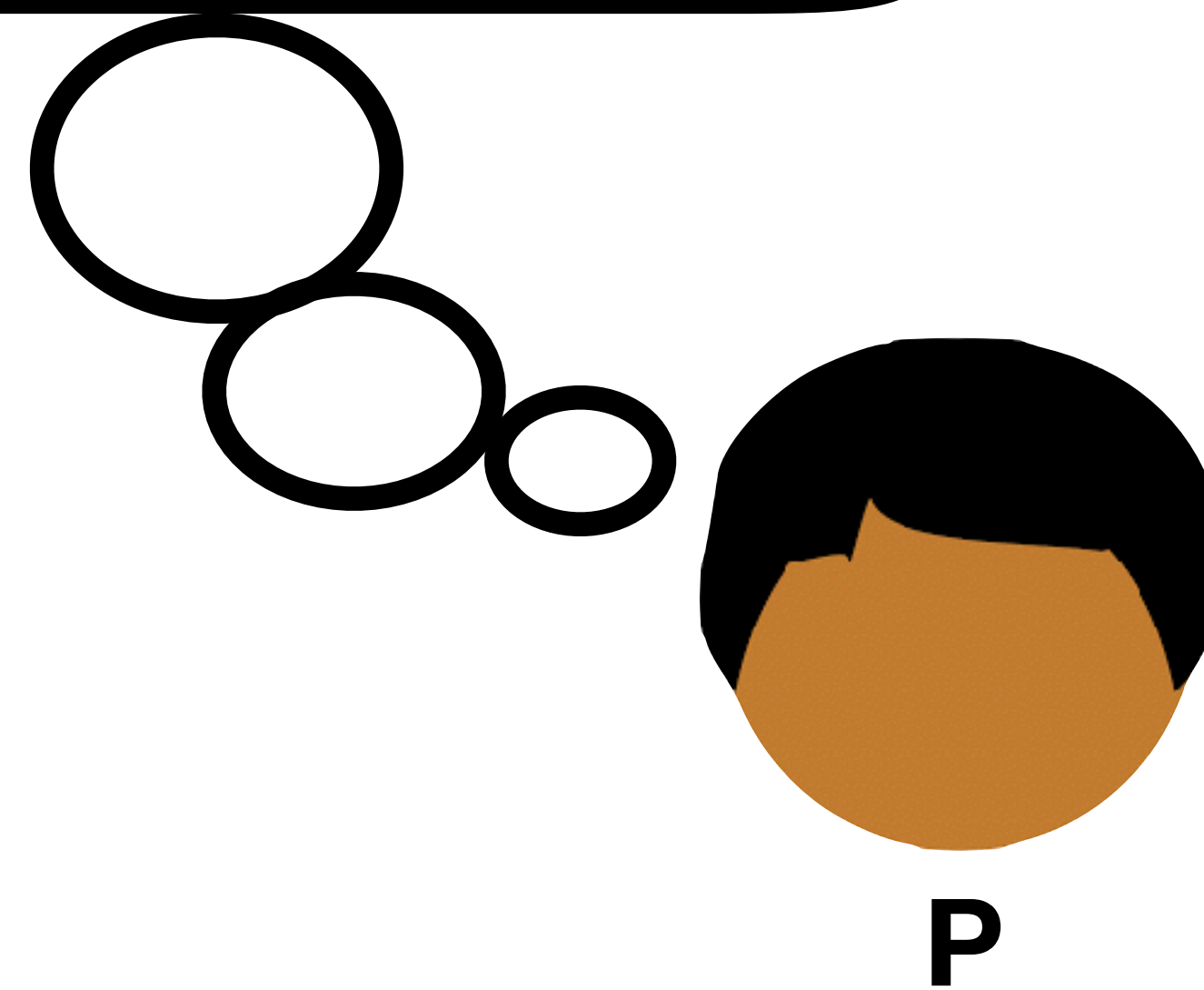
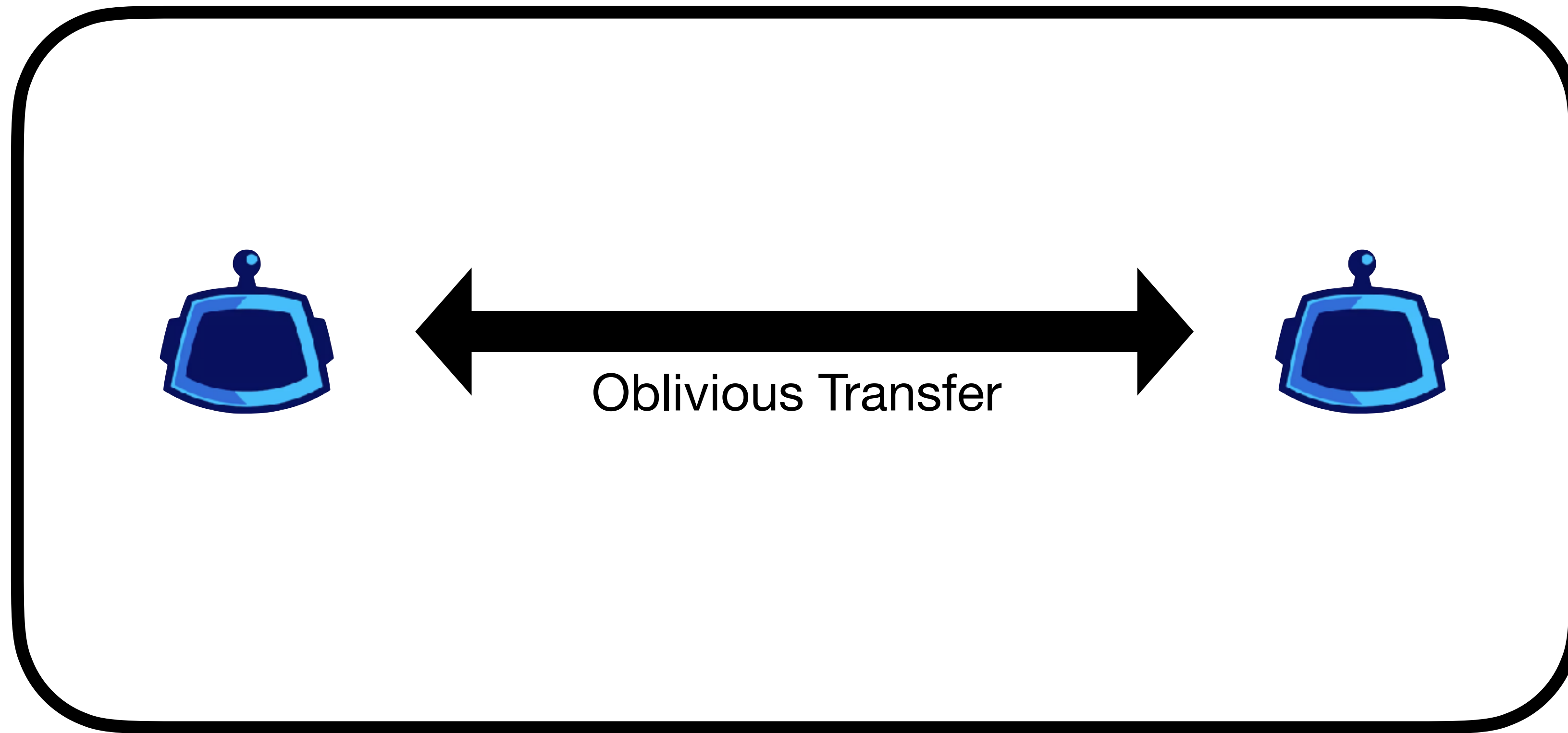


By opening an edge, V has probability at least 1/3 to catch cheating P

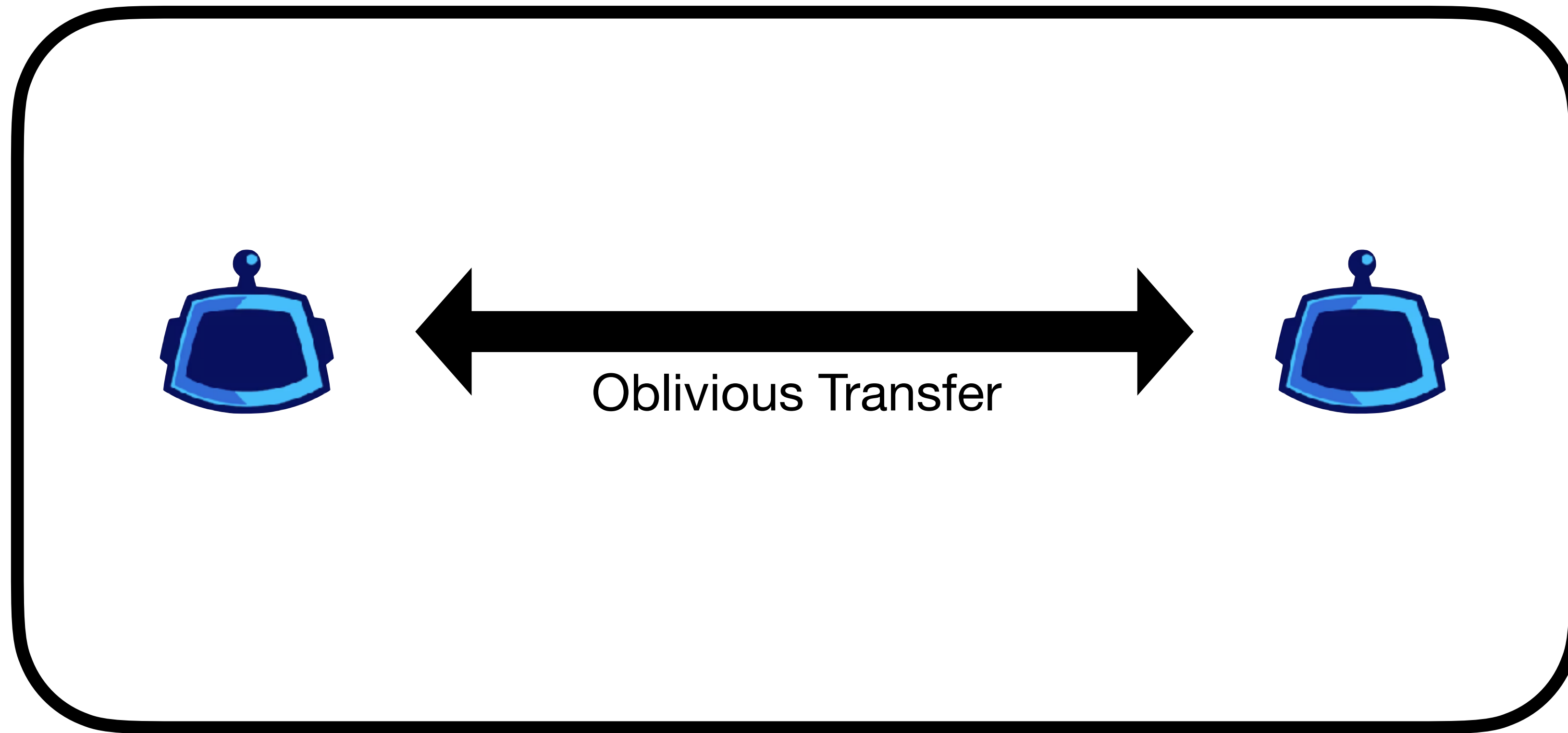
Repeat to obtain desired soundness



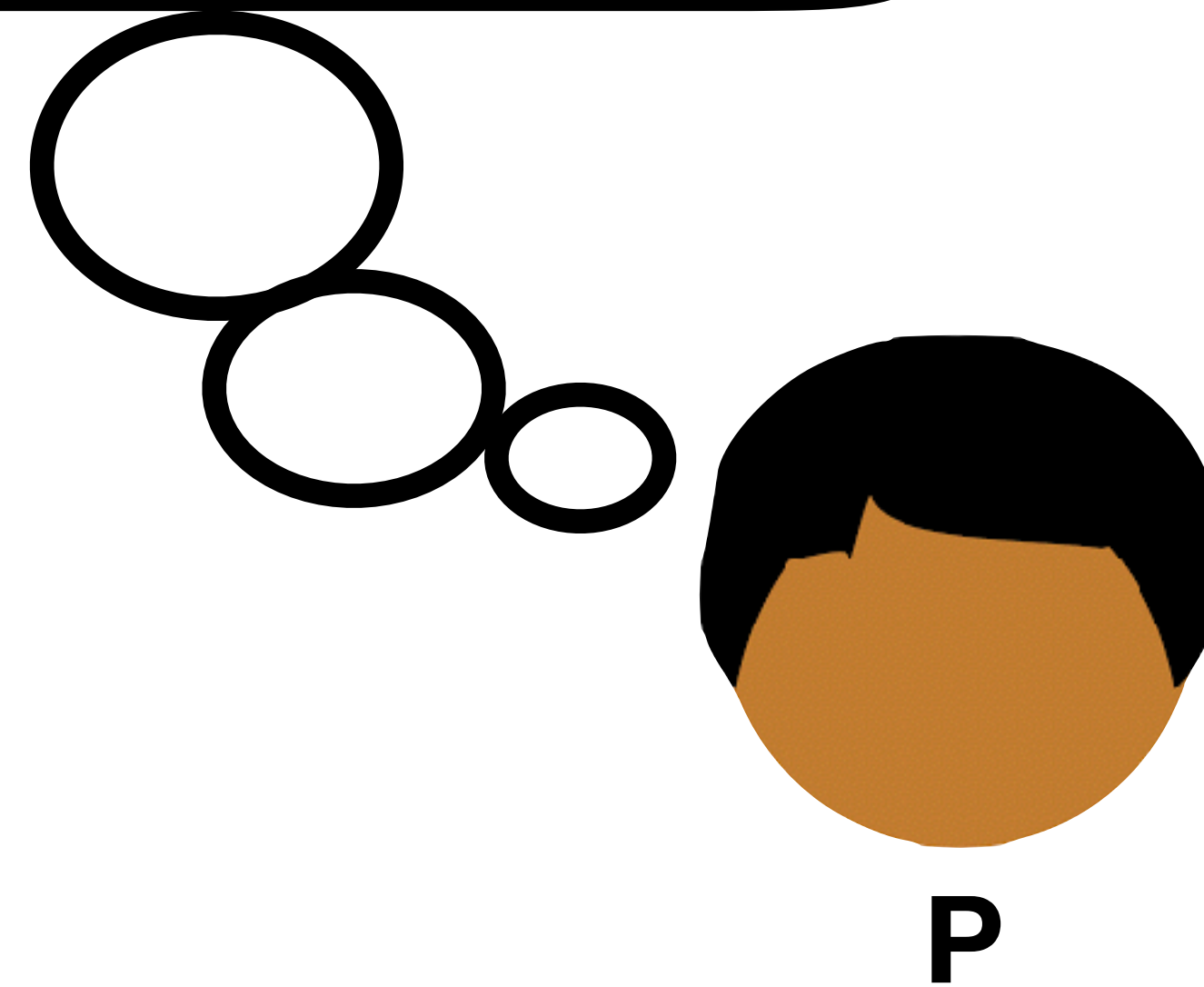
ZK from MPC in the Head



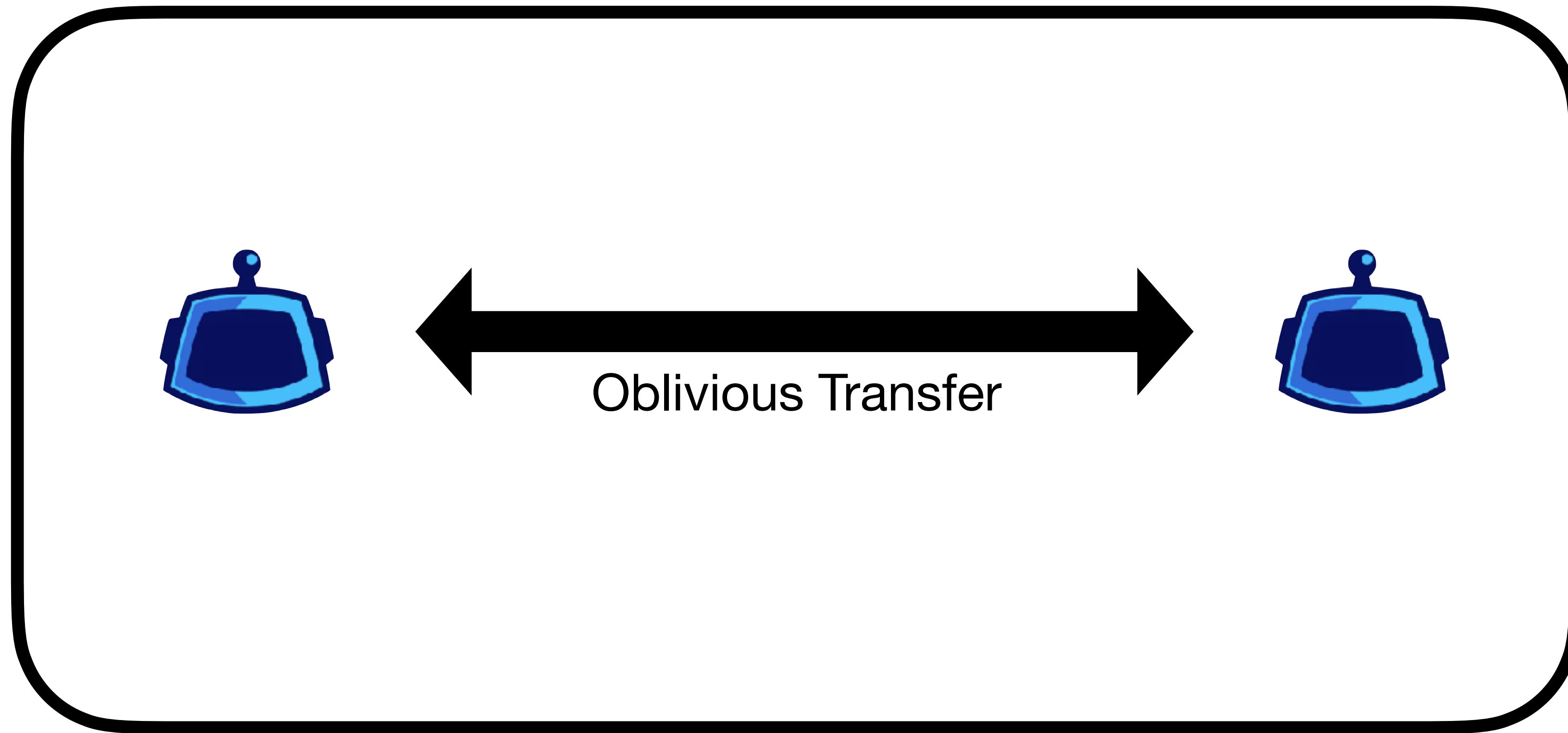
ZK from MPC in the Head



Do virtual parties need to run an actual OT protocol using public key cryptography?

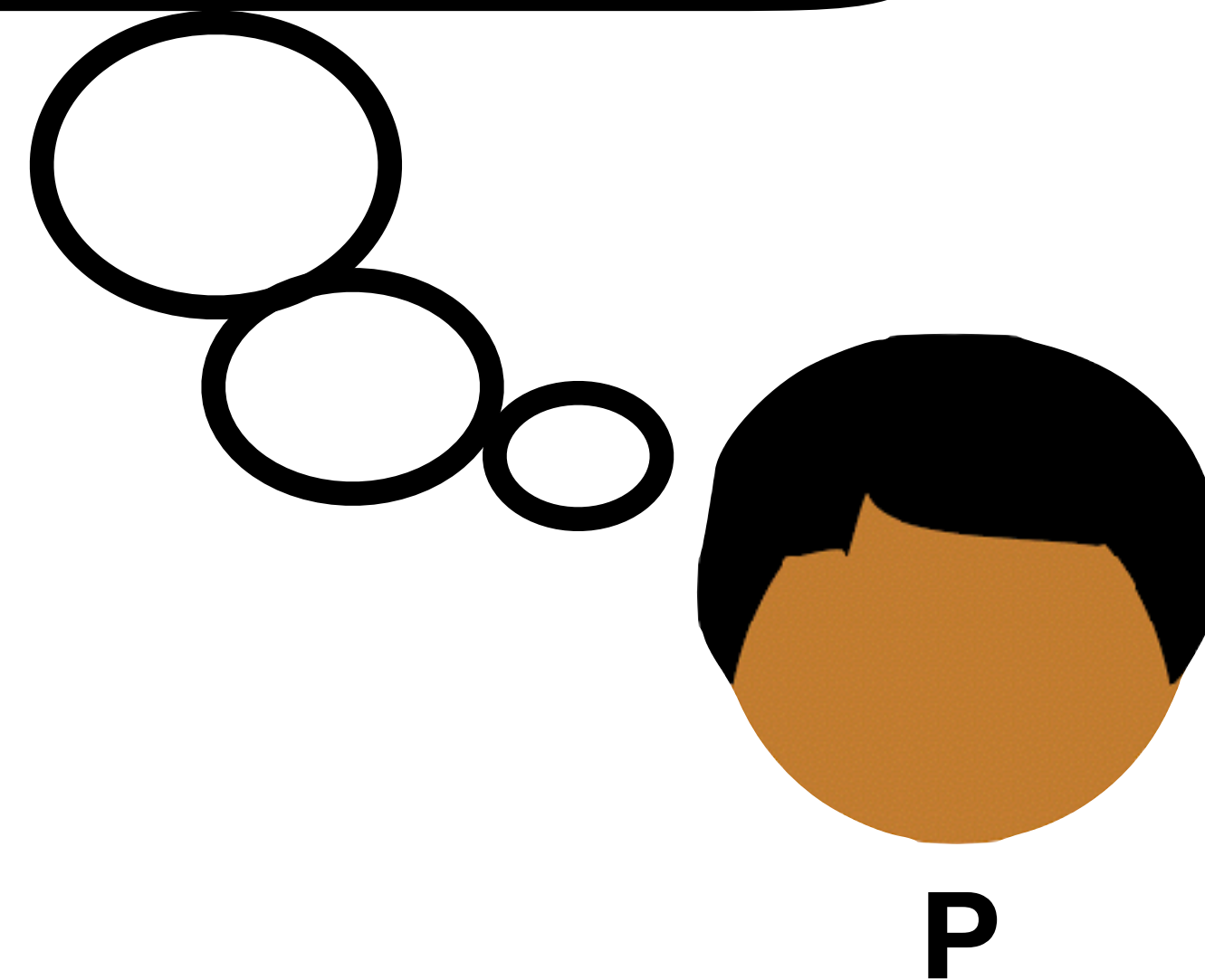


ZK from MPC in the Head

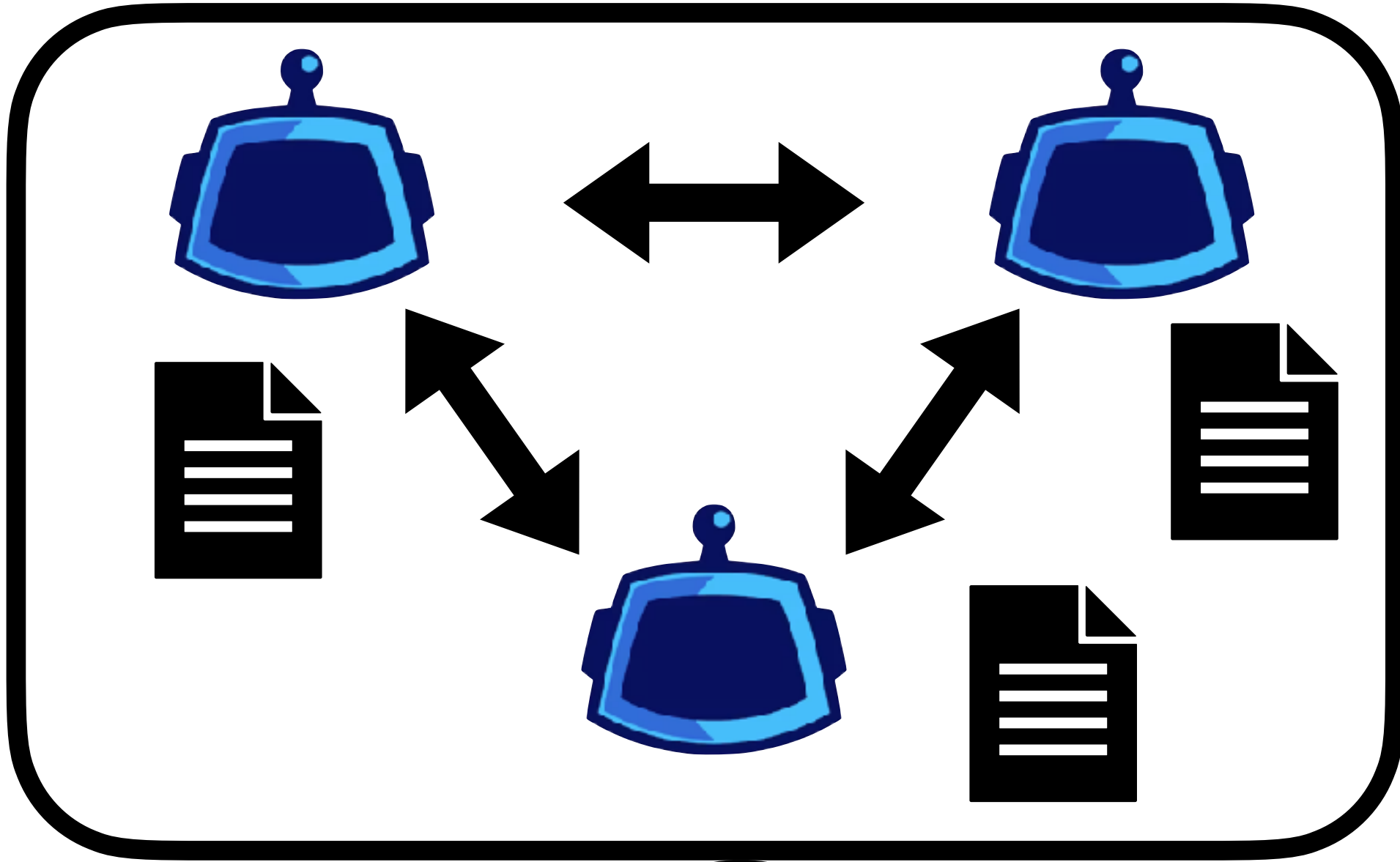


Do virtual parties need to run an actual OT protocol using public key cryptography?

No! P can act as a trusted third party; V just needs to check that the inputs/outputs of OT are consistent



ZK from MPC in the Head

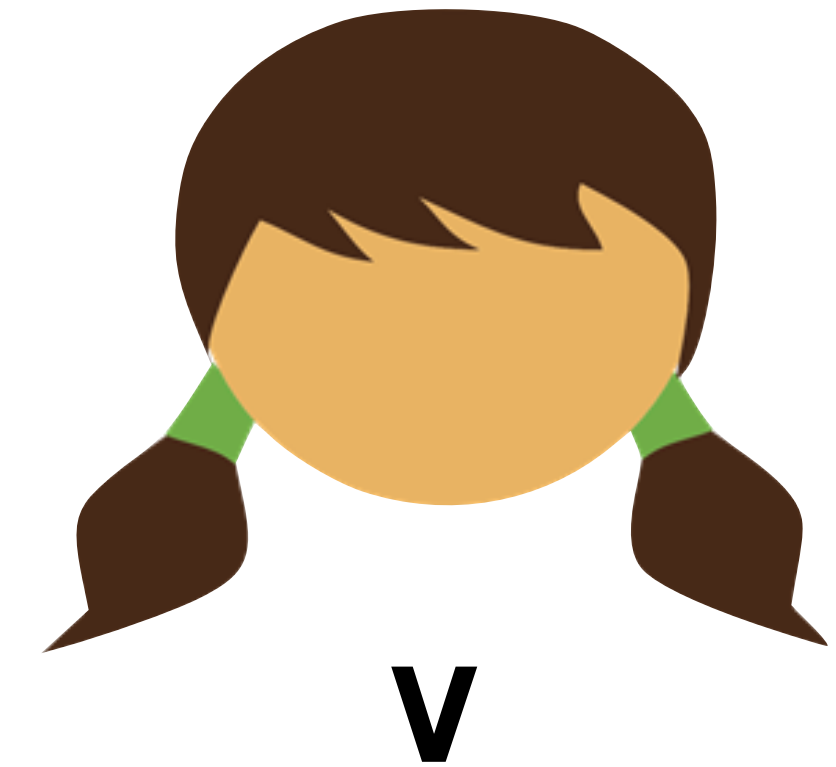
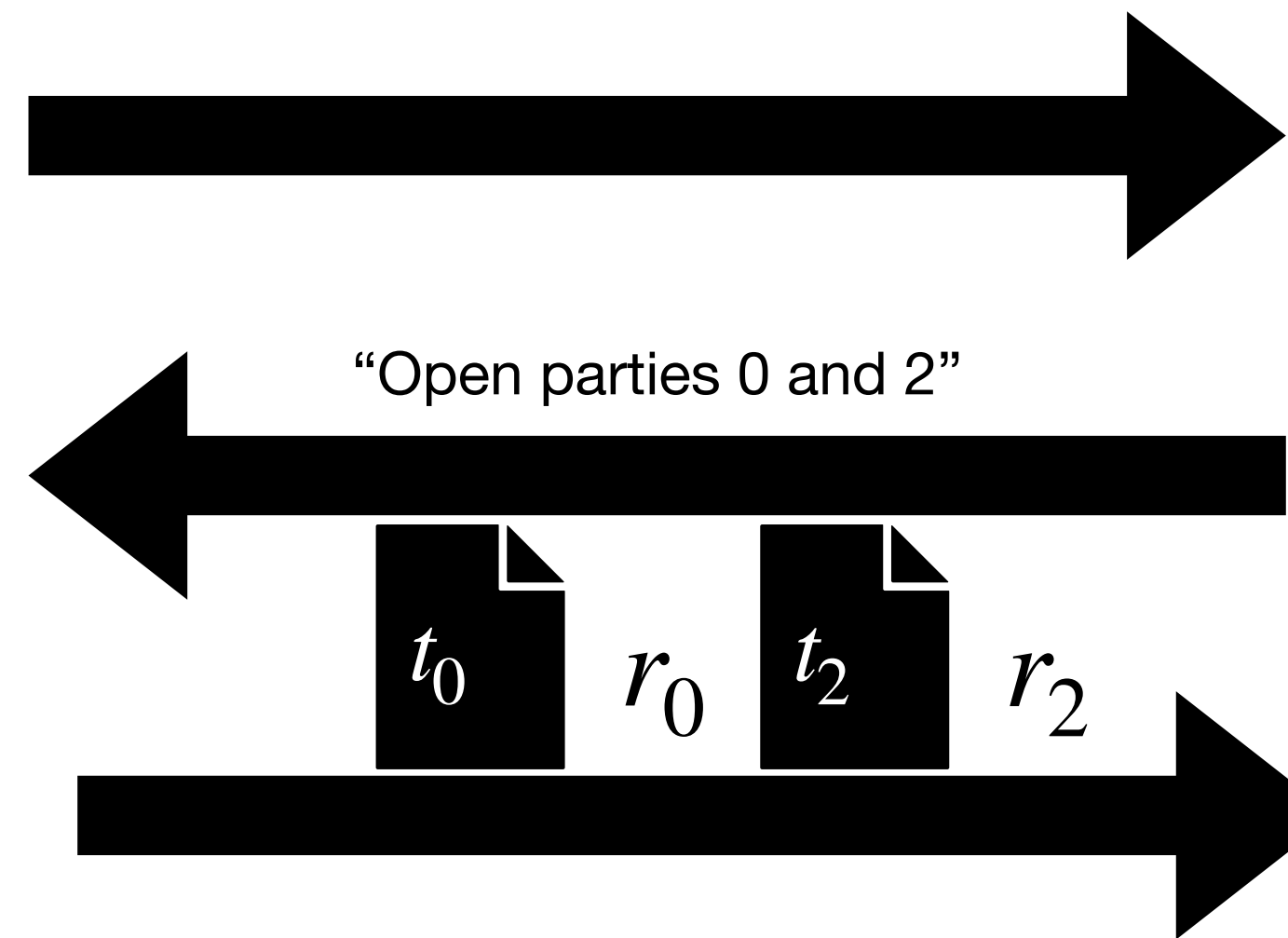
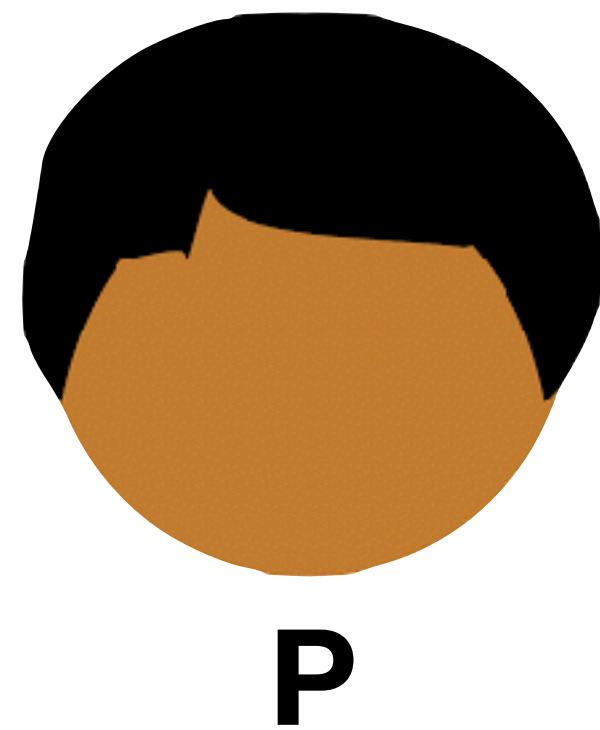
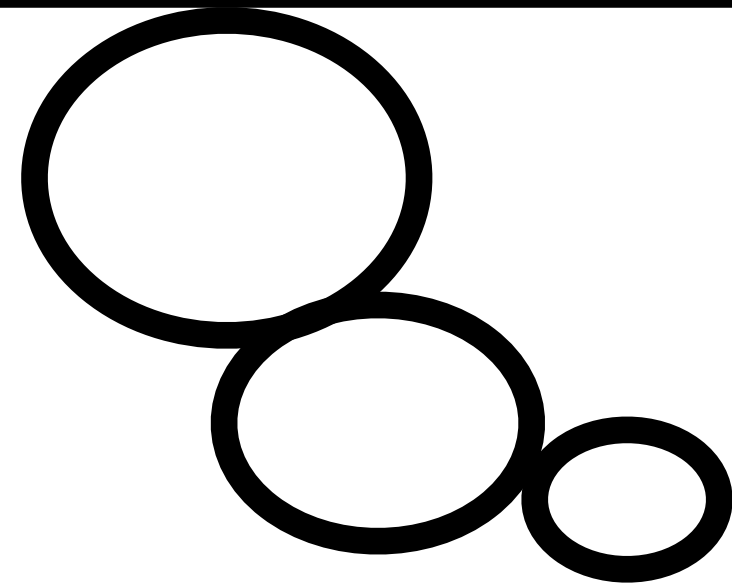


Note, V's random choice is made **after** P commits

$\text{com}(t_0, r_0)$
 $\text{com}(t_1, r_1)$
 $\text{com}(t_2, r_2)$

This is a **public coin** protocol

GC-based protocol is **private coin**



Fiat Shamir Heuristic

Public coin ZK can be made **non-interactive**

How To Prove Yourself: Practical Solutions to Identification and Signature Problems

Amos Fiat and Adi Shamir
Department of Applied Mathematics
The Weizmann Institute of Science
Rehovot 76100, Israel

Abstract.

In this paper we describe simple identification and signature schemes which enable any user to prove his identity and the authenticity of his messages to any other user without shared or public keys. The schemes are provably secure against any known or chosen message attack if factoring is difficult, and typical implementations require only 1% to 4% of the number of modular multiplications required by the RSA scheme. Due to their simplicity, security and speed, these schemes are ideally suited for microprocessor-based devices such as smart cards, personal computers, and remote control systems.

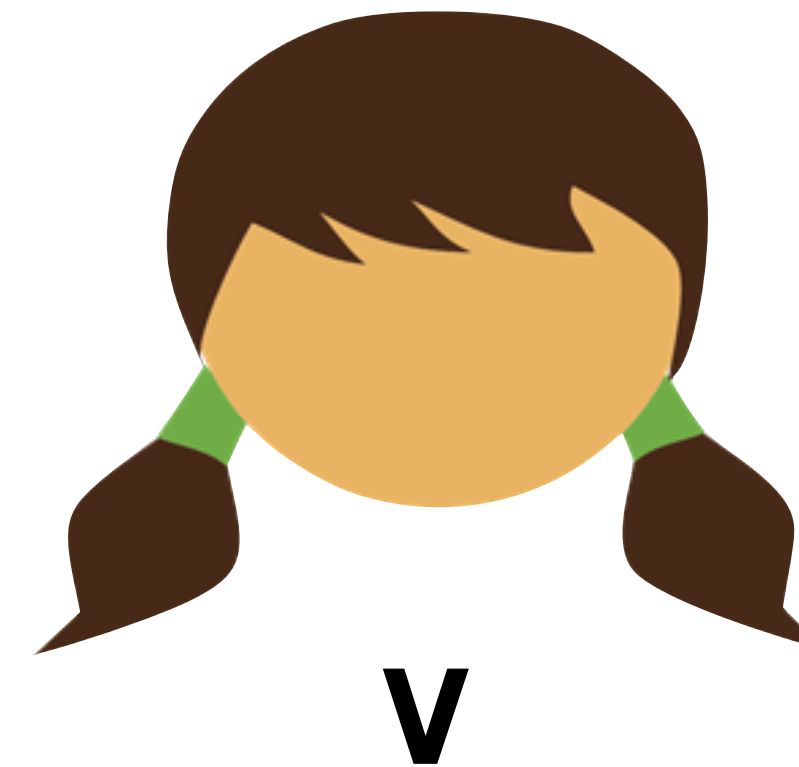
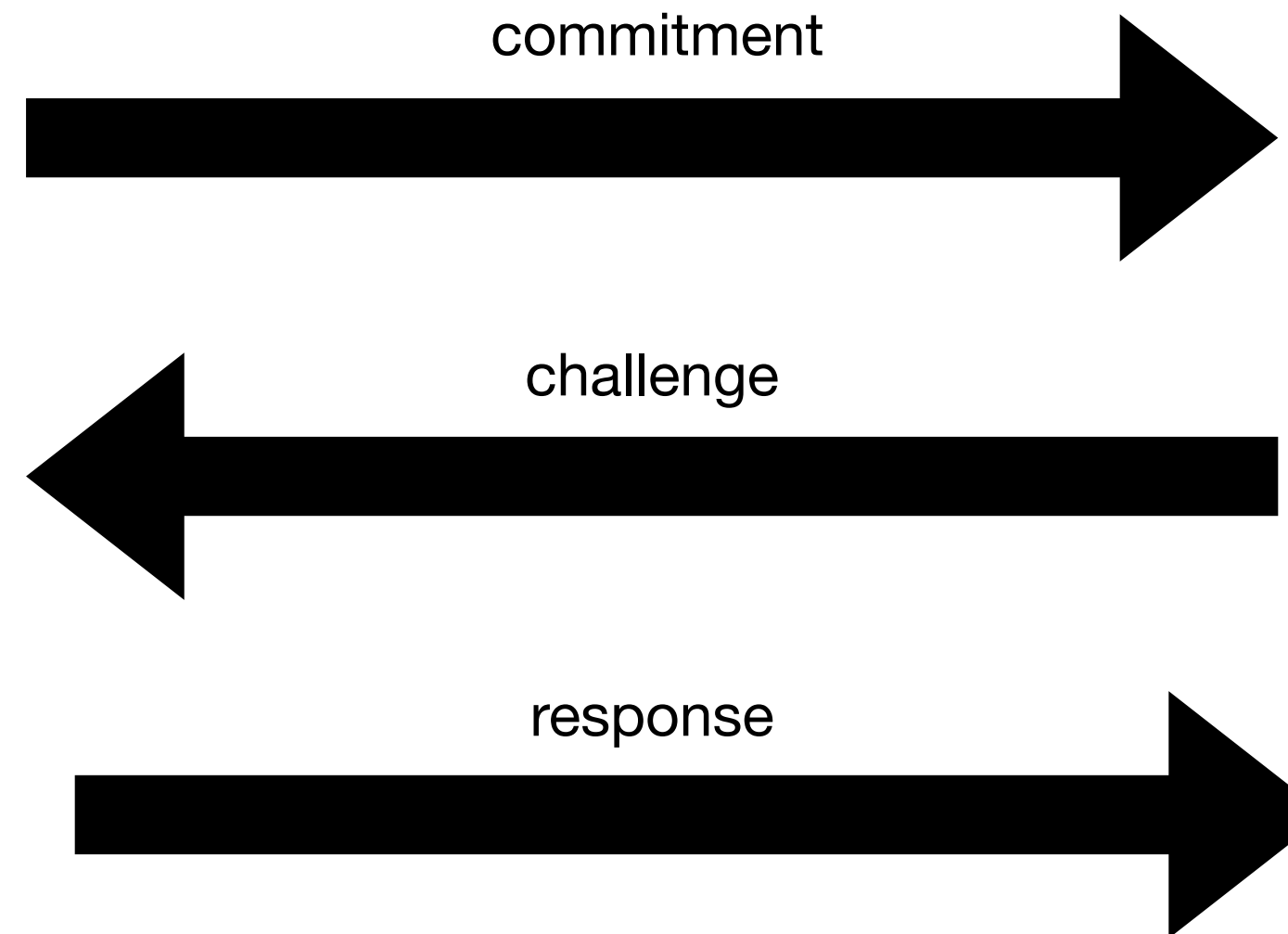
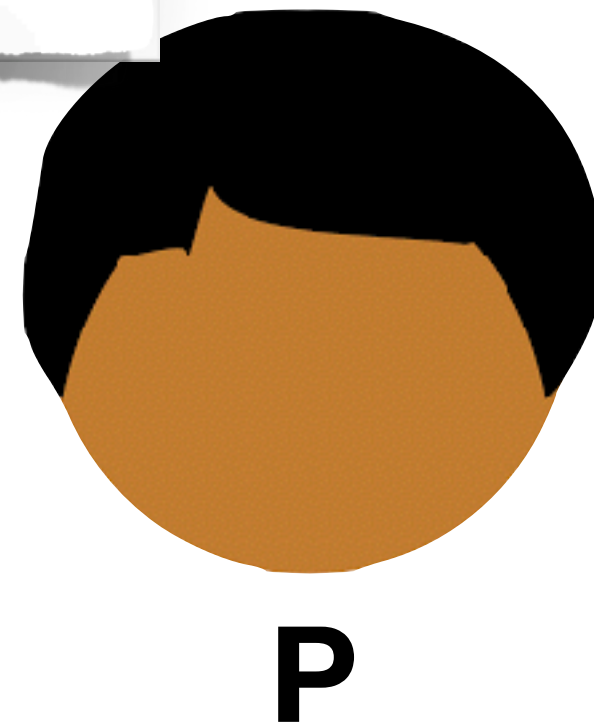
1. Introduction

Creating unforgeable ID cards based on the emerging technology of smart cards is an important problem with numerous commercial and military applications. The problem becomes particularly challenging when the two parties (the prover A and the verifier B) are adversaries, and we want to make it impossible for B to misrepresent himself as A even after he witnesses and verifies arbitrarily many proofs of identity generated by A . Typical applications include passports (which are often inspected and photocopied by hostile governments), credit cards (whose numbers can be copied to blank cards or used over the phone), computer passwords (which are vulnerable to hackers and wire tappers) and military command and control systems (whose terminals may fall into enemy hands). We distinguish between three levels of protection:

- 1) Authentication schemes: A can prove to B that he is A , but someone else cannot prove to B that he is A .
- 2) Identification schemes: A can prove to B that he is A , but B cannot prove to someone else that he is A .
- 3) Signature schemes: A can prove to B that he is A , but B cannot prove even to himself that he is A .

Authentication schemes are useful only against external threats when A and B cooperate. The distinction between identification and signature schemes is subtle, and manifests itself mainly when the proof is interactive and the verifier later wants to prove its existence to a judge: In identification schemes B can create a credible transcript of an imaginary communication by carefully choosing both the questions and the answers in the dialog, while in signature schemes only real communication with A could generate a credible transcript. However, in many commercial and military applications the main problem is to detect forgeries in real time and to deny the service.

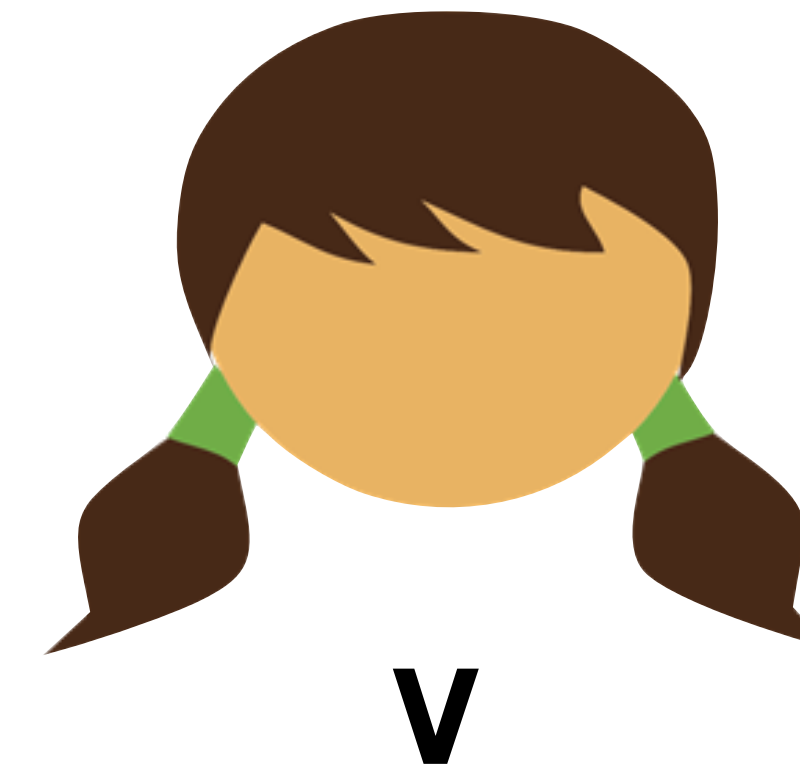
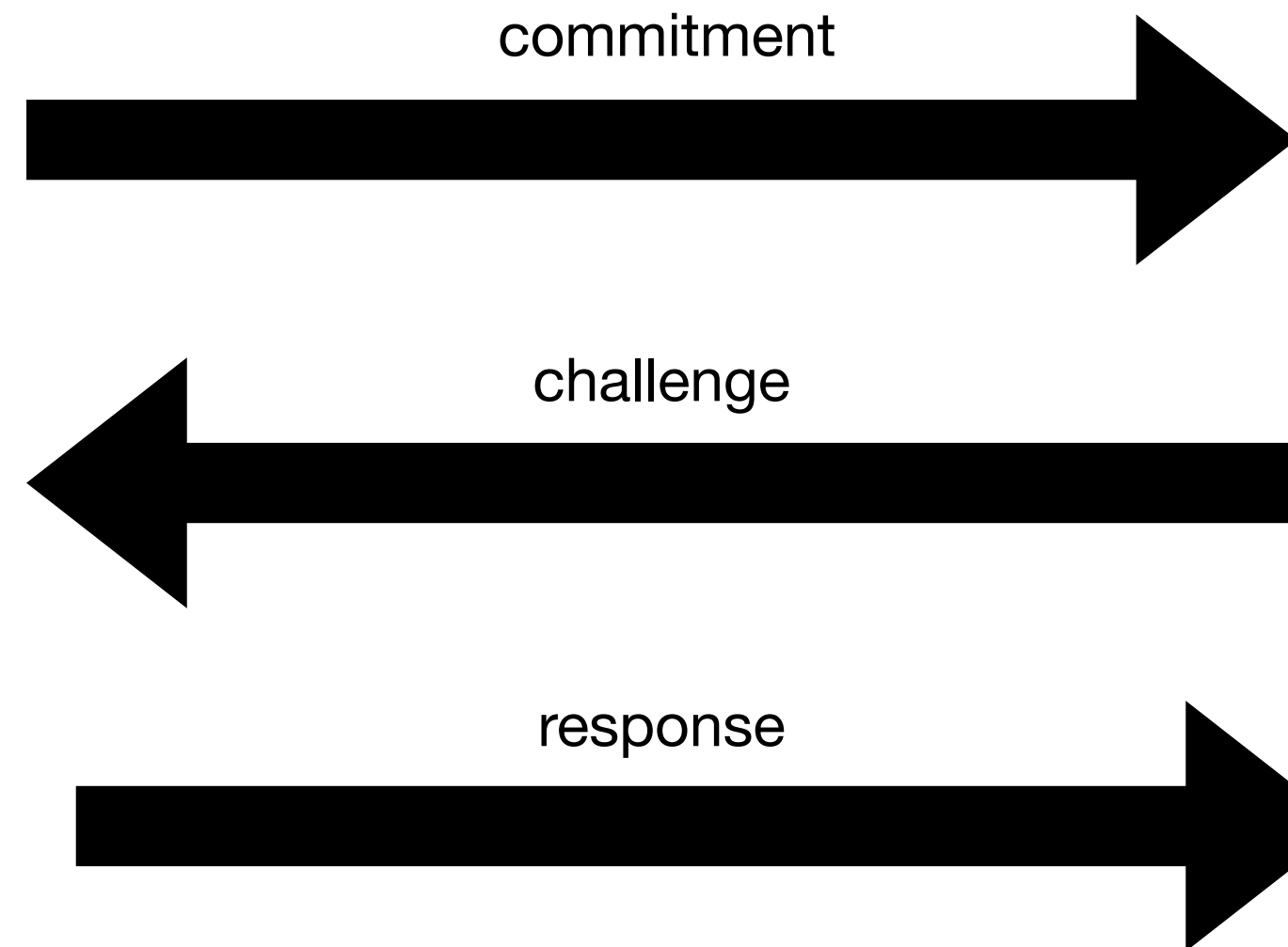
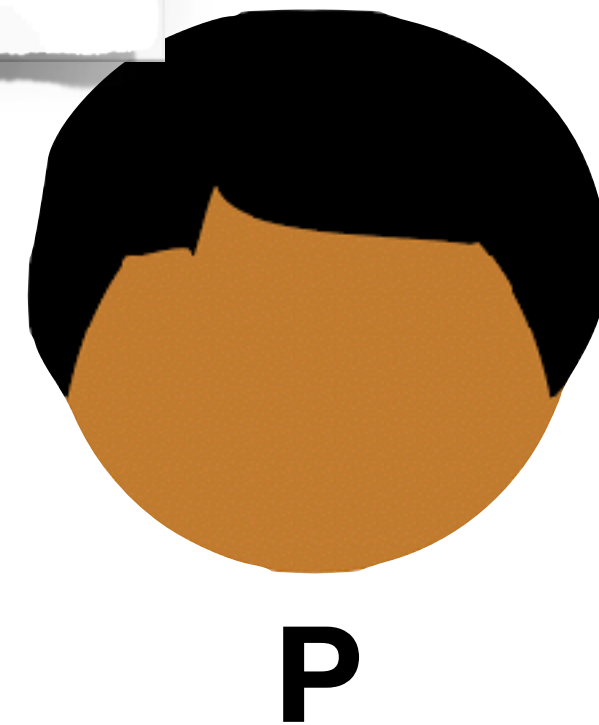
A.M. Delfino (Ed.): Advances in Cryptology - CRYPTO '86, LNCS 263, pp. 186-194, 1987.
© Springer-Verlag Berlin Heidelberg 1987



Fiat Shamir Heuristic

Public coin ZK can be made **non-interactive**

Simple idea: P can choose the challenge itself



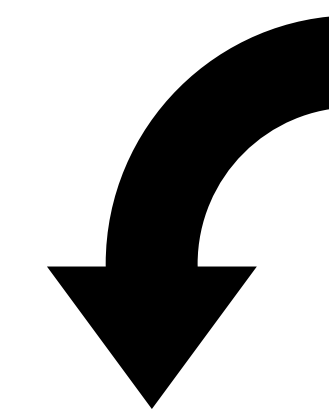
Fiat Shamir Heuristic

Public coin ZK can be made **non-interactive**

Simple idea: P can choose the challenge itself

Cryptographic hash function
(e.g. SHA 256)

Formally, a random oracle

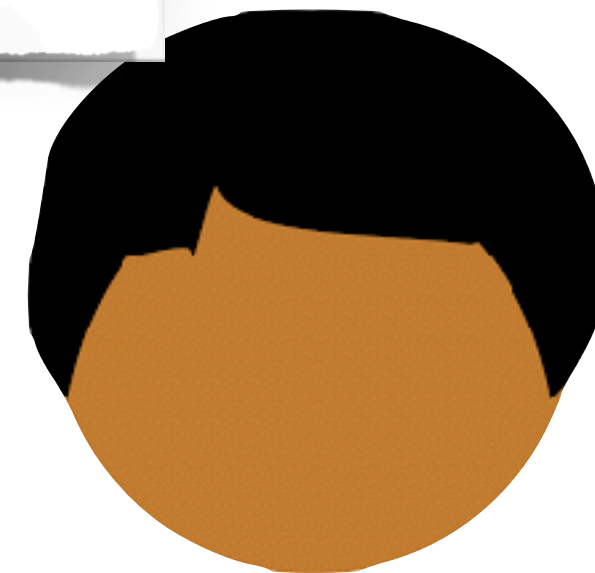


challenge = $H(\text{commitment})$

commitment



response



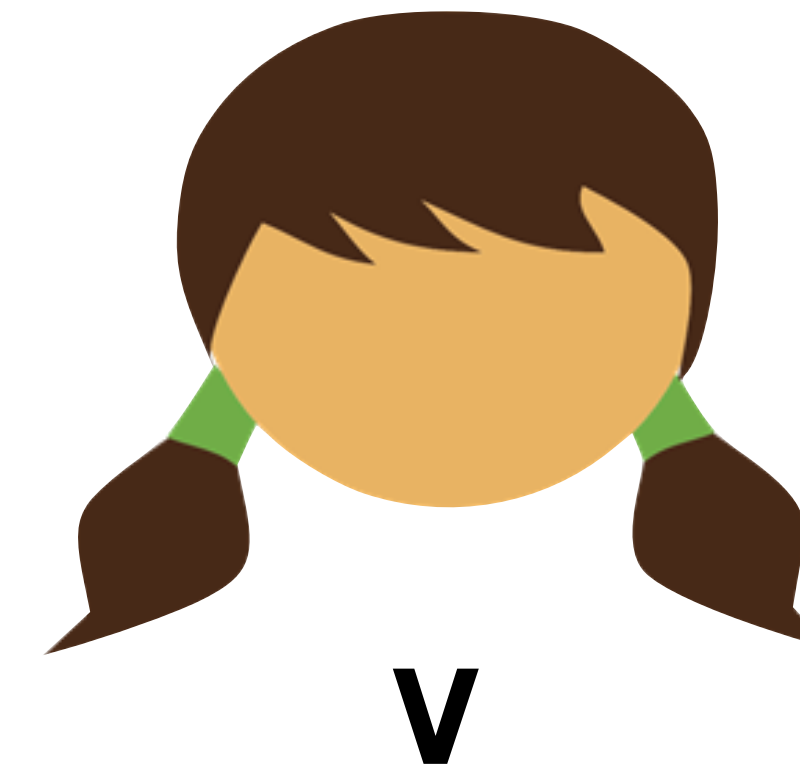
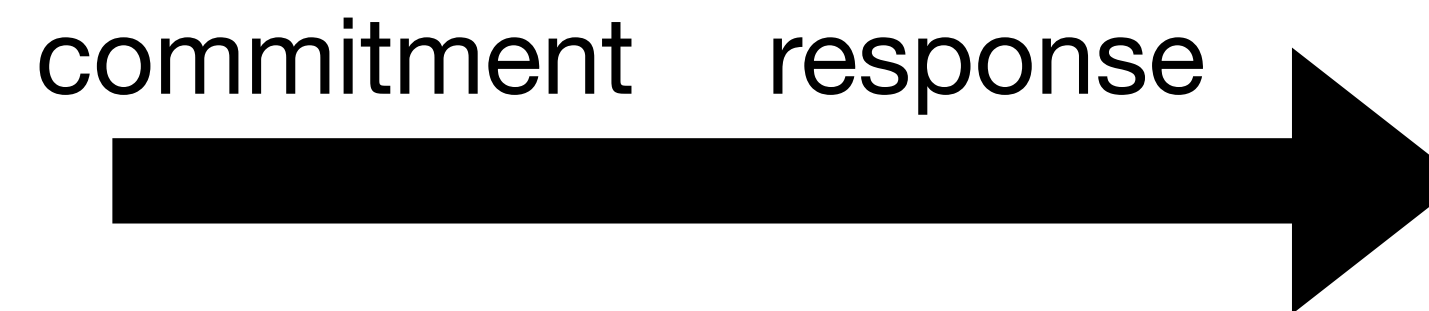
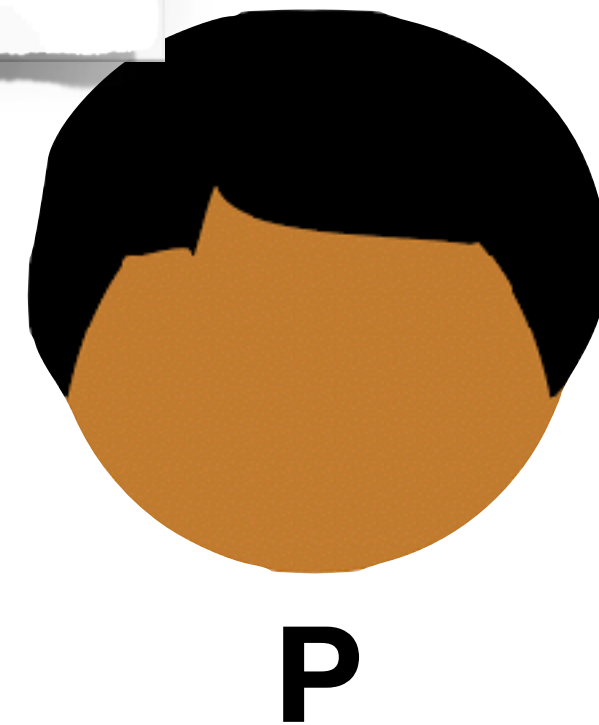
P



Fiat Shamir Heuristic

Public coin ZK can be made **non-interactive**

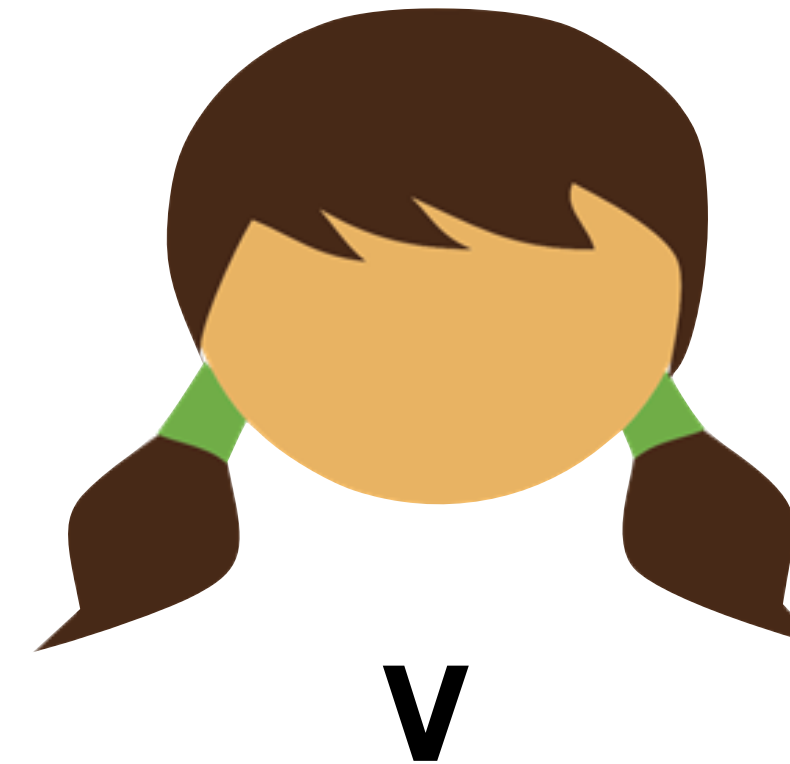
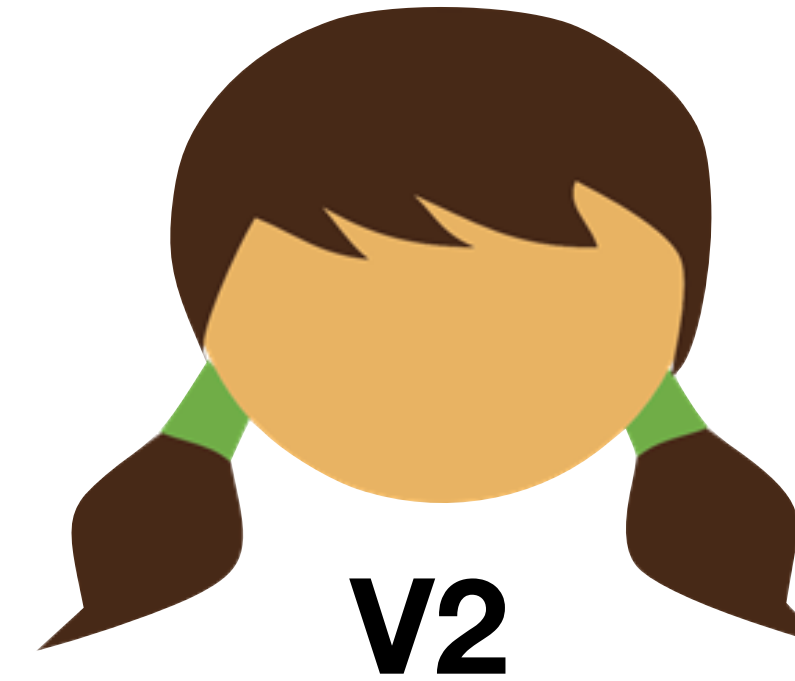
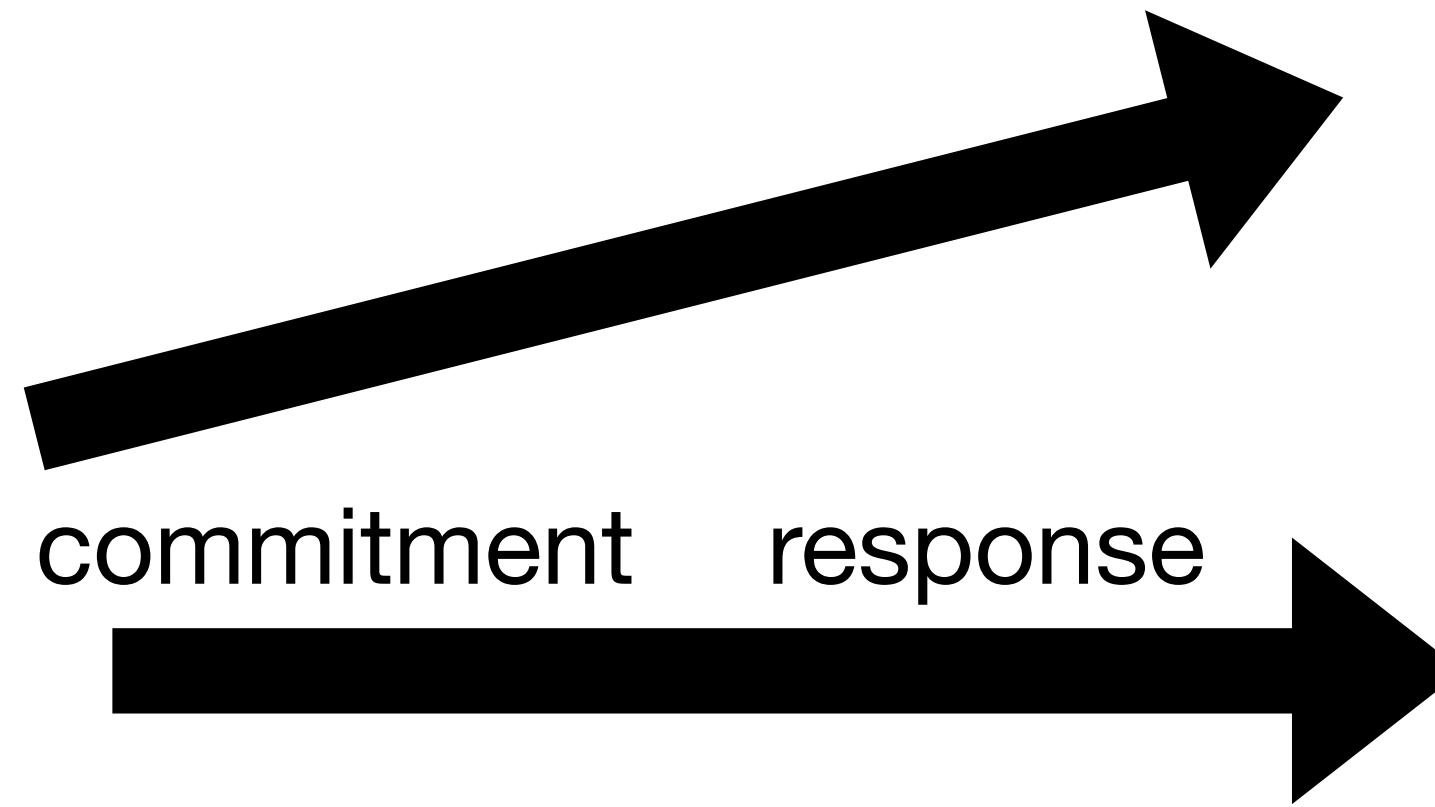
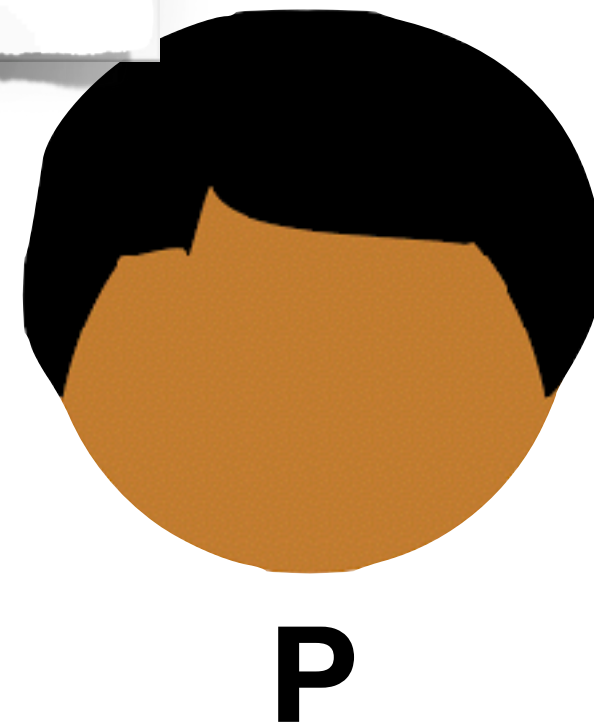
Simple idea: P can choose the challenge itself



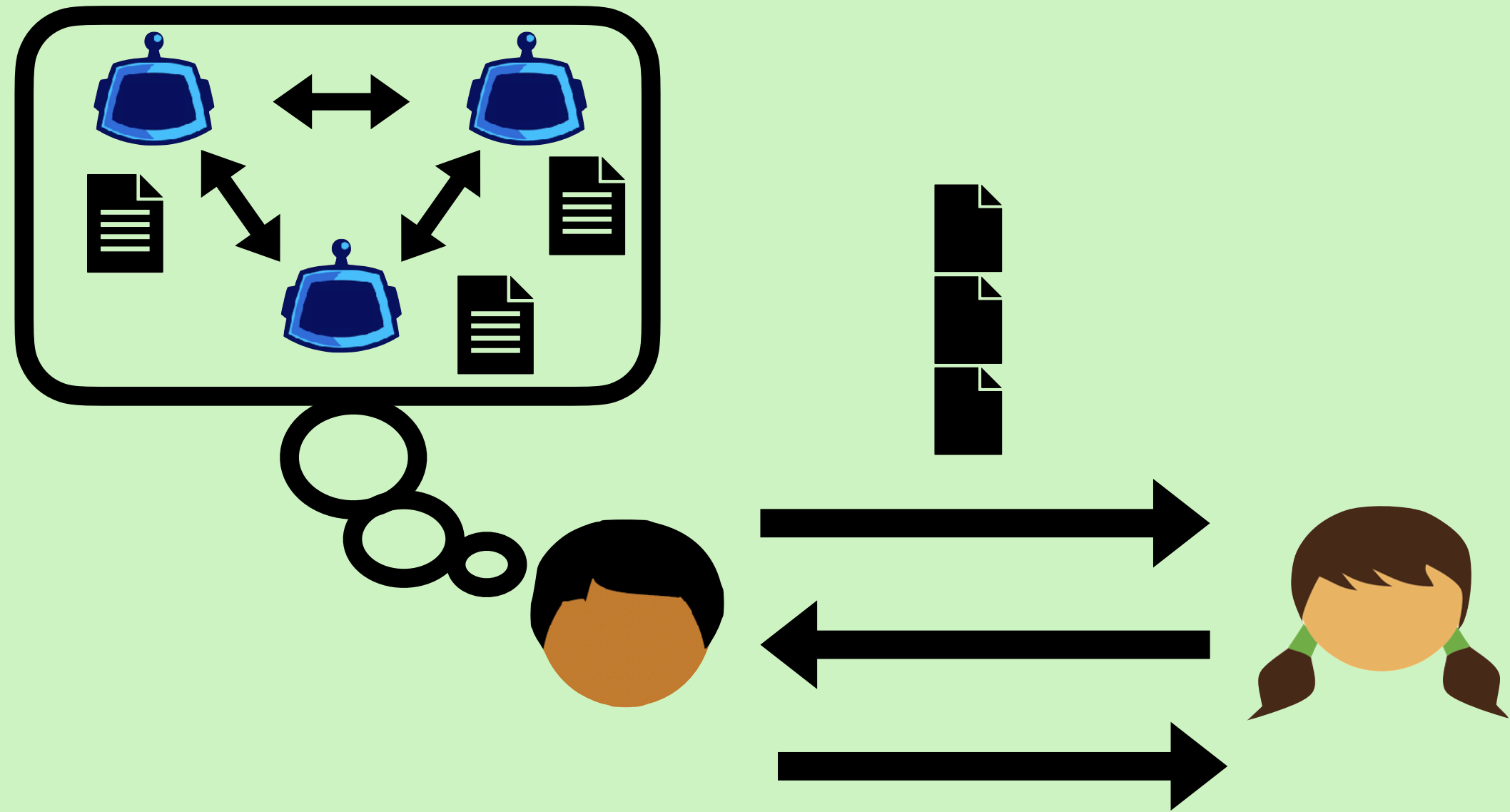
Fiat Shamir Heuristic

Public coin ZK can be made **non-interactive**

Simple idea: P can choose the challenge itself



ZK from MPC in the Head



P runs an MPC protocol in its head to calculate transcripts

V challenges a subset of the transcripts

P opens, and therefore is caught with some probability if cheating

P can commit to multiple repetitions of the protocol to amplify soundness

P can calculate its own challenges using a hash function (Fiat Shamir)

Today's objectives

Review Zero Knowledge Proofs

Construct ZK for circuit satisfiability problem

- From Garbled Circuits
- Via “MPC in the Head”